and $T^2 = \frac{(R+r)^2 - \beta^2}{-s^2}$ so that

Thus we see that as β increases both t and T decrease on account of the minus signs in δt and δT. Also if I were to increase, $\frac{\beta\delta\beta}{t\,(m_1-s_1)^2}$ should be greater than

$\frac{\beta\delta\beta}{T\,(m_1-s_1)^2}$ ie. $\frac{1}{t}$ must be greater than $\frac{1}{T}$ ie.

T should be greater than t and this is so. Hence as β increases t and T decrease but $T-t$ increases ie. as T decreases $T-t$ increases ie. I increases. So, roughly I is taken to be inversely proportional to T so that $I' = \frac{I \times T}{T'}$

If strict inverse proportionality is there, since,

and $T + \delta T = T' = T - \frac{\beta\delta\beta}{T\,(m_1-s_1)^2}$ and therefore

$$\frac{t'}{T'} = \frac{t}{T}\,\frac{\left\{1 - \frac{\beta\delta\beta}{t^2\,(m_1-s_1)^2}\right\}}{\left\{1 - \frac{\beta\delta\beta}{T^2\,(m_1-s_1)^2}\right\}} = \frac{t}{T}\,\frac{\left\{t^2 - \frac{}{(m_1-s_1)^2}\right.}{\left\{T^2 - \frac{}{(m_1-s_1)^2}\right.}$$

t^2 must be roughly equal to T^2 which means $T-t = I$ must be small. In other words the formula of the verse holds good only for small I's or Iṣṭakālas.

In fact this process of rectification of the Bhuja was first given by Brahmagupta in verses 18 and 19 of Sūrya-grahaṇādhikāra and accepted by Srīpati in verse 14. The verse 19 under Lunar eclipses in the Sūrya-siddhānta also seems to indicate this process but uses the words Madhya-Sthiti-khanda and Spaṣta-Sthiti-khanda in an ambiguous sense. Ranganātha in his commentary takes the Madhya-Sthiti-khanda to mean that rectified for variation in β, and Spaṣta Sthiti-khanda to mean that rectified for parallax. Some modern traditional commentators, for example pandit Sitha Ramajha commenting on the verse of the Sūryasiddhānta and the commentator of the verses of Brahma Sphutasiddhānta published under the editorship of Acharya Rama Swarupa Sharma, have ignored the variation in β and confined themselves only to the effect of parallax in longitude.

Yet one more proof could be adduced in this behalf. Let D, be the moment of geocentric conjunction, t the time for a given grāsa in between D and T the moment of first geocentric contact. Then the times D, t and T are to be corrected both for parallax in longitude and that in latitude. Take the geocentric Sthiti-khanda, and geocentric Bhuja as the mean-values T and B and those corrected first for parallax in longitude as the True values T' and B'. Also take the time t corrected for parallax in longitude to be t'. We could write $T' - T = \delta T$, $t' - t = \delta t$ and $B' - B = \delta B$. In the above, consider B expressed in time. Then $D - T = S$ the mean Sthiti-khanda, $D - t = B$ the mean Bhuja, $D' - T' = S'$, the true Sthiti-khanda rectified for parallax in longitude, $D' - t' = B'$ the Bhuja also rectified for the same. The $\delta B = \delta D - \delta t$, $\delta I = \delta D \;\; \delta T$. Then using the proportion " If for $D - T$ we have $\delta D - \delta T$ as the variation what shall we have for $D - t$?" The result

D—T

'—T') _ Mean Bhuja × True Sthiti-khanda
Mean Sthiti-khanda

In other words to obtain the Bhuja rectified for parallax in longitude, we have to multiply the mean Bhuja by the Sthiti-khanda rectified for parallax and divide by the Mean Sthiti-khanda. Similarly to obtain the Sthiti-khanda rectified for the variation in latitude also, take the Sthiti khanda rectified for parallax as the mean and that rectified for variation in β as the True and then by the same procedure, Bhuja rectified for parallax in latitude will be equal to Sthiti-khanda rectified for parallax in latitude multiplied by the Bhuja rectified for parallax in longitude divided by the Sthiti-khanda rectified for parallax in longitude Bhāskara gives the names Sphuta Sthiti-khanda and Sphuta S'araja-Sthiti-khanda to the Sthiti-khanda rectified for parallax in longitude and that rectified further for parallax in latitude. The proof adduced above accords with the statements of Brahmagupta, Srīpati and Bhāskara; Ranganātha bearing in mind Bhāskara's version puts a correct interpretation on verse 19, Lunar eclipses of Sūryasiddhānta. But the usage of the words Madhya and Sphuta in that verse, misled the modern traditional scholars including Burgess. If is to be mentioned here that the word Koti translated as perpendicular by Burgess is misinterpreted by him (see his commentary under verse 19, lunar eclipses) as 'The perpendicular is furnished us in time and the rule supposes it to be stated in the form of the interval between the given moment and that of contact or separation'. This translation goes against the definition of Koti contained in verse 18 just above, since the Koti of Sūryasiddhānta is the Bhuja of Bhāskara and vice versa.

We shall now see why there arose confusion in the minds of many modern commentators including Burgess. The problem mooted by the Sūryasiddhānta in calling for a rectification of the Koti, is to be noted as that when the grāsa g is given and not the Iṣtakāla I. The two formulae for Koti are $\frac{R+r-g^2-\beta^2}{u-v}$ and (T −I) $(u-v)$. When I is given we have to use the latter formula and take T″ in the place of T where T, T′ and T″ are respectively the Sthityardhas (1) Mean, (2) Mean rectified for parallax in latitude and (3) Mean rectified for parallax in longitude. Of course here to arrive at T″, method of successive approximation is to be used as β goes on changing from time to time and there is an inter-play between the simultaneous effects of parallax in longitude and that in latitude.

On the other hand when g is given we have to use the former formula for the Koti and the Koti thus obtained is to be rectified for the variation in β. So Sūryasiddhānta proposes the formula

$\frac{\sqrt{(R+r\ \ g)^2-\beta^2}}{u-v} \times \frac{T'}{T''}$ meaning thereby that the correction for the variation in β is more important because the formula is in terms of β and not the other formula. This formulation is approximate but adopted for the sake of ease. Otherwise from the grāsa, I is to be obtained and the other formula could be used which method is more laborious.

Bhaskāra's Correction of Brahmagupta's Statement

Verses 1, 2 *&* 3. The statement of Brahmagupta namely that the arc of the Moon's Dṛk-kṣepa will be obtained by the sum or difference of that of the Sun with the latitude of the Vitribha, I (Bhāskara) do not accept. I shall give the reason why. In a place where the latitude is 24°. when the longitudes of the Sun, the Moon and the Node are all 180°, at the time of Sun-rise, the ecliptic occupies the position of the prime-vertical. The Moon will not leave the ecliptic even though depressed by parallax in longitude. Thus there is no parallax in latitude.. The Vitribha then being in the zenith, and its latitude being $4\frac{1}{2}°$, the Dṛk-kṣepa of the Moon according to Brahmagupta's formula will be $4\frac{1}{2}°$ and therefore the parallax in latitude obtained by the Dṛk kṣepa will be

$\frac{790'-35''}{15} \times \frac{\text{H sin } 4\frac{1}{2}°}{3438} = \frac{52'-42''}{3438}\, 270 = 4'-8''$ which is not the case actually.

Comm. Having thus shown the flaw in Brahmagupta's approximate formula, Bhāskara proceeds to show how that formula could be justified in a particular way. Brahmagupta assumed the Moon's orbit to be the ecliptic because at the moment of an eclipse, the latitude of the Moon is very small so that he might be taken to be on the ecliptic. In figure 101 let the Ecliptic coincide with the prime-vertical ZE. Let EV be the Moon's orbit where V is the Vitribha of the Moon's orbit. The arc of the Sun's Dṛk-kṣepa is here zero and that of the Moon's Dṛk-kṣepa is ZV which is the sum of the arc of the Sun's Dṛk kṣepa namely zero and the latitule of the Moon's Vitribha namely ZV, since the pole of the ecliptic now coincides with the south point S, which means that ZV is the latitude of V.

Let VV′ be the nati which is equal to AB, since V′A is the so-called Vikṣepa Sadṛs'amandala or the deflected position of VE on account of parallax in latitude. The four minutes of parallax in latitude is now VV′=AB. This parallax is obtained because, Bhāskara argues, the nati obtained by Brahmagupta is there because he took VE to be the Ecliptic and so obtained that nati with respect to VE. Now, Bhāskara says, this is to be corrected by the difference of the Moon's latitudes the original one and that obtained after the Moon is deflected by the parallax in longitude. This difference is 0—AB=BA. Correcting AB with BA, the result is AB+BA=0 so that ultimately there is no parallax in latitude ie. no nati at all, as should be the case.

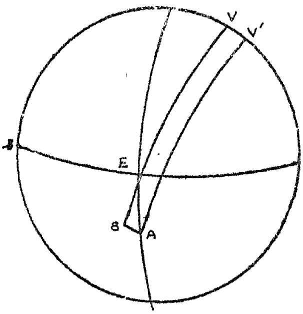


Fig. 101

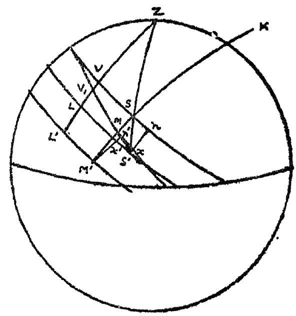


Fig. 102

Or, Bhāskara's argument could be better illustrated from figure 102. Let VS be the ecliptic where V is the Sun's Vitribha and S, the Sun. Let V′M be the Moon's orbit called Vikṣepamandala where V′ is taken to be the Moon's Vitribha and M the Moon. Let S′, M′ be the deflected positions of the Sun and the Moon on account of parallax. ZV′=ZV+VV′ = the arc of the Sun's Dṛk-kṣepa + the latitude at the point V called Vitribhalagna—Bāna, as formulated by Brahmagupta that ZV′ is roughly equal to the Moon's Dṛk-kṣepa-Dhanus. (Strictly speaking ZV′ ought to be perpendicular to the Moon's orbit V′M, if V′ were to be the Vitribha of the Moon; but, as the latitude is small, the error is negligible). Let LS′ and L′M′ be the so-called Krānti-Sadṛs′a-mandala and Vikṣepa Sadṛs′amandala or parallel drawn to the ecliptic and the Moon's orbit through the deflected positions S′ and M′ of the Sun and the Moon. S′n is the Sun's parallax in latitude ie. Nati. Similarly Brahmagupta took M′n' to be the nati of the Moon as stated by Bhāskara. The error committed will be therefore M′n'—S′n = (M′x'+$x'n'$)—(S′x+xn)=M′x' − xn) cancelling $x'n'$ and S′n which are roughly equal. Also xn could be roughly taken to be equal to MS. Hence the relative parallax is equal to $M'x' - MS =$ Difference of the latitudes of the Moon in his original and deflected positions respectively. Hence $S'n = M'n' - (M'x' - MS)$ = Brahmagupta's nati+correction of the difference of the latitudes reversely effected, as stated by Bhāskara.

In fact, Bhāskara has misread Brahmagupta's correct procedure, since the latter sought the relative parallax of the Sun and the Moon.

Instead of adding the latitude at V to the zenith-distance of V (the arc of the Sun's Dṛk-kṣepa) which implies additional computation of that latitude, as an approximate procedure, MS is added to ZV to get ZV' as an alternate procedure as mentioned by Bhāskara in the course of the commentary.

Whereas Brahmagupta was seeking relative parallax, Bhāskara misread that Brahmagupta took $M'n'$ as the nati and did not effect the correction of $(M'x - MS)$ to obtain $S'n$, which Bhāskara took to be the Sphuta-nati. Not effecting the above correction is interpreted as neglecting it since the Moon's latitude during the course of an eclipse is small.

GRAHACCHĀYĀDHIKĀRA

Verse 1. The orbital inclinations of the planets.

The inclinations of the orbits of Mars, Mercury, Jupiter, Venus and Saturn to the ecliptic are respectively 110, 152, 76, 136 and 130 minutes of arc. The nodes of Venus and Mercury get rectified by adding their respective S'īghra anomalies to their values obtained originally.

Comm. The values given above are said to be the mean values. Those given for the superior planets namely Mars, Jupiter and Saturn approximately accord with their modern values. Bhāskara says that these values pertain to that moment of observation, when the S'īghra anomaly is equal to $90+\frac{1}{2}$ H $\sin^{-1} a$ where a is Hsine of the maximum S'īghraphala. This is quite in order because when the S'īghra anomaly assumes the said value, the true planet is at the point of intersection of the deferent and the eccentric, which means that the planet is equidistant from E_1 as well as E_2 (vide fig. 103). Identifying E_1 to be the earth's centre and E_2 to be that of the Sun in the case of the superior planets, the mean latitude of the planet observed will be the same as that observed either from the earth or from the Sun. Hence in the case of the superior planets, the maximum latitudes of the planets observed accord with the geocentric as well as heliocentric observation. These are taken to be the mean values of the maximum latitudes.

In the case of the inferior planets, it will be clear why the modern values of 7° and 3°–24′ for Mercury and Venus are far higher than the Hindu values namely 152′ and 136′. Since the mean planet in this case is taken to be the Sun, the linear values of the latitude observed from E and S, the centres of the Earth and the Sun respectively will be in the ratio SP/ES (vide fig. 104).

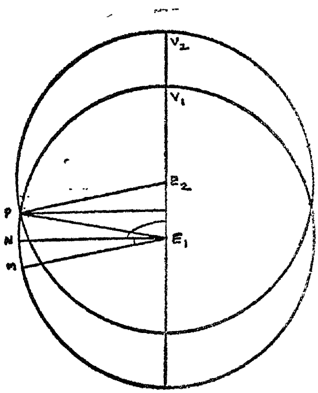


Fig. 103

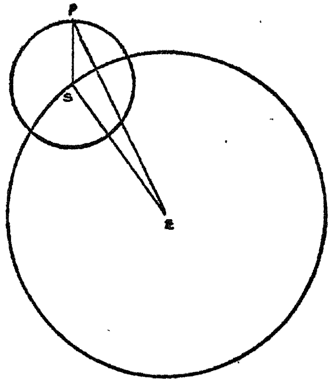


Fig. 104

In the case of Mercury this ratio is and that in the case of Venus is $\frac{7}{10}$. Hence the values 420×4 and $\frac{204 \times 7}{10}$ ie. 168′ and 142′ roughly accord with the Hindu values. In other words the Hindu values are geocentric and the modern heliocentric.

Bhāskara adds that the nodes of Mercury and Venus as computed previously are to be increased by the S'ighra anomaly to obtain the actual longitude of the node from which the latitude is to be computed. This directive is a beautiful example to show implied heliocentric motion. Let the convex angle ASN be the heliocentric longitude of the node measured negatively as the node has a negative motion along the ecliptic. Adding the heliocentric S'ighra anomaly to this longitude of the node means convex angle $ASN + \widehat{USP} = 360° + \widehat{NSP} - \widehat{ASU} = \widehat{NSP} - \widehat{ASU}$. Now add the longitude of the planet ie. here $\widehat{ASU}$ ($=\widehat{AES}$) to obtain the argument from which the latitude of P the planet is to be computed. The result is NSP as should be.

Computing the latitude as indicated in the next verse, by the formula $\frac{\text{H sin NSP} \times \beta \times \text{R}}{\text{R} \times \text{R}}$ where β is the maximum latitude cited above in the respective cases, multiplied by *R* and divided by K indicates that the linear magnitude of the latitude will be increased or decreased according as K is smaller or greater than R.

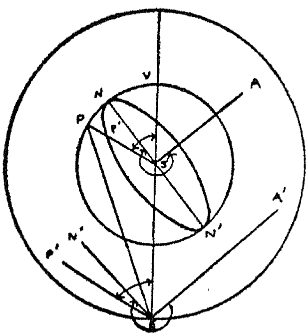


Fig. 105

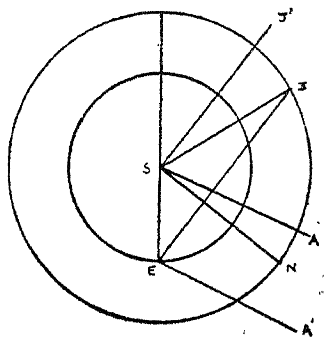


Fig. 106

(a) In this context, we are to throw light on some moot points. Bhāskara says "मन्दस्फुटो ग्रहः स्वशीघ्रप्रतिमण्डले ', which tantamounts to saying that the node is situated in the heliocentric orbit "स्वशीघ्रप्रतिमण्डल". The Vimandala or the planet's orbit, taking for example the inner orbit of P in fig. 105, does not exactly lie in the plane of the ecliptic as shown in that figure but is in an inclined position cutting the coplanar inner orbit in N and N′, as shown by the ellipse NPN′ where NP = NP′. The argument to calculate the latitude is NP′ and the formula is $\frac{\text{H sin NP}'}{\text{R}} \times \beta$ to give the heliocentric linear latitude. To obtain its geocentric linear value, we have to multiply by R

(b) In the case of the superior planets, Bhāskara mentions in the Golādhyāya "पातोऽथवा शीघ्रफलं विलोमं कृत्वा स्फुटात् तेन युतात शरोऽत:" verse 22 (Golabandhādhikāra). This also clearly indicates that the orbits of the superior planets are also heliocentric, for, otherwise, there is no purpose of subtracting the S'īghraphala from the True position of the planet. The purpose in doing so is to obtain the heliocentric longitude of the planet (vide fig. 106).

True geocentric longitude of J is $A'EJ = ASJ'$.

Subtracting the S'īghraphala $EJS = JEJ'$ from ASJ', we have $\widehat{ASJ}$ the heliocentric longitude. Adding the retrograde longitude of N (तेन युतात् as cited above in verse 22) means adding ASN to $\widehat{ASJ}$ which gives $\widehat{NSJ}$. This is the argument to calculate the latitude of J.

(c) Bhāskara alludes to a confusion in the mind of Chaturvedāchārya in this context while commenting upon verses 23, 24 in Golabandhādhikāra (Golādhyāya). Chaturvedācharya commenting on Brahmagupta's words (verse 10 Grahayukti-adikāra ch. 9, Brahma Sphutasiddhānta) exclaims. "The latitude of Mercury and Venus will be the same as what they are at the point of S'īghroccha; correctness of the result alone is proof; no other reason could be adduced!" Bhāskara clears his misconception in the following words. "The number of sidereal revolutions of the nodes of Mercury and Venus mentioned in the chapter on mean motion, are to be increased by the number of the sidereal revolutions of the S'ighrocchas of Mercury and Venus, as mentioned by Mādhava in his work "Siddhānta-chudāmani". This means that the S'īghra anomaly is to be added to the position of the node obtained by the smaller number of revolutions given in the chapter on mean motion.

The misconception in the mind of Chaturvedāchārya as well as the wrong notion in the minds of Mādhava and even Bhāskara in construing that the number of revolutions of the nodes are to be increased by the number of revolutions of the S'īghrocchas, are due to the fact that it was overlooked that the position of S'īghrocchas with respect to Mercury and Venus are given by their heliocentric longitudes. In other words as is mentioned by the verse "ज्ञशुक्रयोः ग्रहः सूर्यः भवेत्तौ शीघ्रनामकौ" the S'īghrocchas of Mercury and Venus are no other than the heliocentric positions of those planets. Thus the exclamation of Chaturvedāchārya cited above arose out of thinking that the *S'īghrocchas differ from the planets*, whereas they are the same heliocentrically, though they differ geocentrically. Heliocentrically the planets longitude is (vide fig. 105) $\widehat{ASP}$ which is equal to A/EP/ geocentrically. The geocentric longitude of the planet is on the other hand A/EP differing from the above, though both the heliocentric and geocentric longitudes point to the same planet. Bhāskara, no doubt, gave a correct procedure but missed to identify the S'ighroccha and the heliocentric position of the planet. In this context, the reader is referred to the author's 'peculiar concept of S'īghroccha in Hindu astronomy published in the journal of Oriental Research of the S. V. University Vol. XIV, Part 2, Dec. 1971.

Verse 2. The Hsine of the arc of the planet's orbit Vimandala intercepted between the nearer node and the planet multiplied by the maximum latitude of the planet cited, and divided by the S'īghrakarṇa gives the latitude of the planet at the given place.

Comm. From the formula akin to that which gives the declination of a point on the ecliptic namely $\sin \delta = \sin \lambda \sin \omega$ or in the Hindu form $H \sin \delta = \frac{H \sin \lambda \; H \sin \omega}{R}$,

$H \sin \beta = \frac{H \sin \lambda \; H \sin i}{R}$ where β is the latitude required, i the maximum latitude and λ the arc of the planetary orbit intercepted between the nearer node and the planet. Since β and i are small H sin β and H sin i could be taken to be β and i. Hence we have $\beta = \frac{i \times H \sin \lambda}{R}$ Since this value is had at the distance of the S'īghrakarṇa, its value at a distance of R should be given by

$$\beta = \frac{i \, H \sin \lambda}{R} \times \frac{R}{K} = \frac{i \, H \sin \lambda}{R} \text{ as formulated.}$$

3. $\sqrt{R^2 - H \sin^2 v}$ is called Yaṣṭi where v is Āyanavalana. The latitude β of the planet multiplied by Yaṣṭi and divided by R or multiplied by H cos δ where δ is the declination of the point whose longitude is $90 + \lambda$, λ being that of the planet and divided by R gives the value of the rectified latitude which could be added to the declination of the foot of the latitude to give what we say the modern declination of the planet.

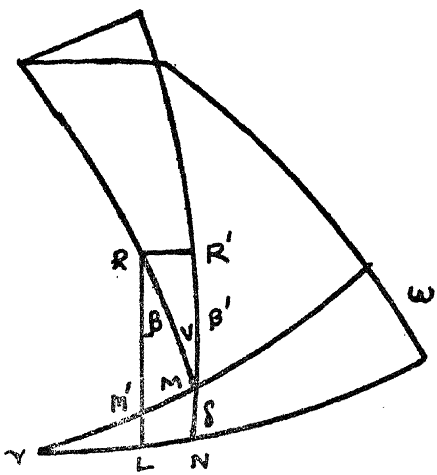


Fig. 107

Comm. Let rM (fig. 107) be the ecliptic and rN the celestial equator whose poles are K and P respectively. Let R be a celestial body whose latitude is β and whose modern declination is RL. Let M be the foot of the latitude circle and let δ be the declination of M.' The word Krānti in Hindu astronomy is applied to connote the declination of a point on the ecliptic alone and not of any other point like R. RL is called *Sphuta Krānti* which is equal to $R'N = R'M + MN$. RM is called *Vikṣēpa* and

Sphuta Vikṣēpa. KMP is called the Āyanavalana at the point M of the ecliptic. Produce MP to P' where $PP' = \delta$ so that $MK = MP = 90°$. Hence $\hat{P'}$ ~~is a~~ right angle and $KP' = K\hat{M}P' = v =$ Āyanavalana. From the spherical triangle KPP', $\cos \omega = \cos v \cos \delta$. Draw perpendicular RR' on MP so that $MR' = \beta' =$ Sphuta Vikṣēpa $= \beta \cos v$

But $H \cos v = \dfrac{\quad}{R}$

$$\therefore \ \beta' = \frac{\text{Yaṣti} \times \beta}{R}$$ which is to be added to δ to obtain the Sphutakrānti $R'N$ or RL, the modern declination.

Note (1) One Mukhopādhyāya, in his thesis 'The Hindu nakṣatras' submitted to the Calcutta University, mistook RM' to be the Sphutavikṣēpa instead of $R'M$ and hastily remarked that Bhāskara was wrong in making RM' less than RM.

Note (2) $\hat{v}$ shown in the fig. 107, is called Sthanīya-valana or valana at the point M which is considered to be place of the planet 'Sthāna' on the ecliptic. $K\hat{R}A$, on the other hand is called Bimbīya-valana or valana at the bimba or disc of the planet.

Note (3) $\hat{v}$ could be obtained from the spherical triangle KMP where $KM = 90°$, $PM = 90 - \delta$, using the formula $\cos \omega = \sin (90-\delta) \cos v$ or in the Hindu form $H \cos v = \frac{R\ H \cos \omega}{H \cos \delta}$. In this case $\beta' = \frac{\beta\ H \cos \omega}{H \cos}$

Or again noting that $\hat{MKP}=90-\lambda$ where λ is the longitude of R, we could use 'Inner side Inner angle formula' with respect to the triangle KMP, which gives $0 = \sin 90 \cot \delta - \sin (90-\lambda) \cot v$ or $\cot v = \frac{\cot \omega}{\cos \lambda}$

or $\tan v = \cos \lambda \tan \omega$. But this formula implies the tangent functions which were not used by the Hindu astronomers.

Similarly using the elements $\overline{90-\lambda}$, 90°, v, and $90-\delta$ of the same triangle KMP, another formula could be got for v. Or again noting that KPM$=90+\alpha$, we could yet get more formulae where α is the Right ascention of M.

Note (4) Thus far we have used modern formulae. Let us now see as to how Bhāskara derives his formula. He takes the triangle MKP′ (fig. 107) wherein MK=90°, $' = v$ and $\hat{P'}=90$. From fig. 108, the Hsine of MK is

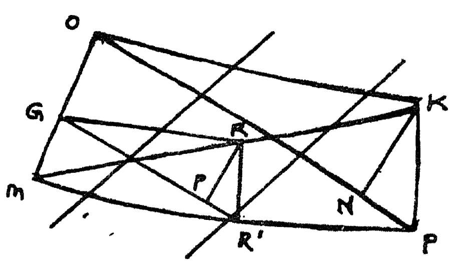


Fig. 108

KO equal to R where 'O' is the centre of the sphere and the Hsine of KP is KN so that it is the Āyanavalanajyā, Hence is the centre of the sphere and the Hsine of KP is KN so that it is the Āyanavalanajyā. Hence

$^{2}=OK^{2}-KN^{2} \quad \therefore \; ON = \text{Yaṣṭi} = \sqrt{R^{2} - \text{Āyanavalanajyā}}$ = Āyanavalanakoṭijyā. From the similarity of the triangles OKN and GRF,

$$\frac{OK}{ON} = \frac{GR}{GF} \quad \therefore \; GF = \frac{GR \times ON}{OK} = \frac{H \sin MR \times \text{Yaṣṭi}}{R}$$

GR could be taken equal to MR, the latter being small and GF could be taken to be equal to GR′ ie. H sin MR′ and so equal to MR′.

$$\therefore \quad MR' = \frac{MR \times \text{Yaṣṭi}}{R}$$

Adding MR′ to the declination of M, we get the modern declination of R′ ie. the Sphuṭakrānti of R.

Note (5) In fact the spherical triangle MKP is just like the spherical triangle *r*E♋ (fig.). In the place of the paramakrānti E♋, we have the Āyanavalana KP (fig. 108), and in the place of Dyujyā ♋L (fig. 19) we have Yaṣṭi.

Note (6) An alternative is given in the verse for this namely $MR' = \frac{MR \times H \cos \delta'}{R}$ where δ' is the nation of a point whose longitude is $\overline{90+\lambda}$. In words we have to prove that $H \cos v = H \cos \delta'$ or $v = \delta'$ (of a point whose longitude is $90+\lambda$. Since declination is given by the formula $H \sin \delta = \frac{H \sin \lambda \times H \sin \omega}{R}$

Putting $90+\lambda$ for λ, $H \sin \delta' = \frac{H \cos \lambda \times H \sin \omega}{R}$

But from triangle MKP, $\frac{\sin v}{\sin \omega} = \frac{\sin 90-\lambda}{\sin 90-\delta} = \frac{\cos \lambda}{\cos \delta}$

$$\therefore \quad \sin v = \frac{\sin \omega \cos \lambda}{\cos \delta} \text{ or } H \sin v = \frac{H \sin \omega \times H \cos \lambda}{H \cos \delta} \quad \text{II}$$

Comparing I and II, v would be equal to δ' provided $H \cos \delta = R$. This is accepted as an approximation, as δ is generally small.

This approximate formula is given by Sūryasiddhānta in the context of Āyana Dṛk-karma in verse 10, ch. 7, where v is assumed to be equal to δ' defined above.

Verses 4 and 6. The process called Āyana Dṛk-karma.

The Āyana Dṛk-karma correction measured in minutes is obtained by multiplying Āyanavalana by the unrectified celestial latitude, and divided by H cos δ and then multiplied by 1800 and divided by the rising time of the Rās'i which is occupied by the planet. Or again it is obtained approximately by the product of the Āyanavalana and the unrectified celestial latitude divided by the Yaṣṭi.

This Āyana Dṛk-karma correction is negative if the hemisphere and the latitude have the same direction.

On symbols, Āyana Dṛk-karma correction =

$$\frac{\text{Āyanavalana} \times \beta \times 1800}{H \cos \delta \times T} \text{ or approximately equal to } \frac{\beta \times \text{Āyanavalana}}{\text{Yaṣṭi } (= H \cos v)}.$$

Comm. Let G be the position of the planet when the foot of the planet's secondary (called Grahasthāna) to the Ecliptic namely A is rising, where rCA is the Ecliptic. Let rML be the celestial Equator. Let P and K be the poles of the celestial Equator and Ecliptic respectively. The arc AC of the Ecliptic intercepted between A and the declination circle of G expressed in minutes represents the आयनकलाः formulated in the verse. To find the magnitude of AC, the first formula mentioned above envisages finding

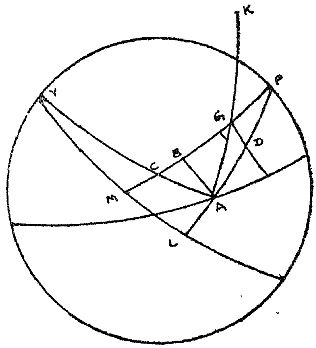


Fig. 109

it through the magnitude of ML the corresponding Equatorial arc which is itself found through finding the magnitude of AB. AB expressed in minutes will be equal to the number of asus taken by the declination circle of G namely PGB to have traversed from the position PDA to PGB which is the time taken by the planet G to be at its position from the moment of its rising at the Equatorial horizon namely PDA, D being then its position. When G was at D, K, the pole of the Ecliptic should have been itself rising on the Equatorial horizon. Thus it is sought to find the time expressed in asus taken by K from its moment of rising on the Equatorial horizon to its present position at K. When Bhāskara mentions in the commentary under the verse that "

by the word क्षितिज, he had at the back of his mind

ie. Equatorial horizon and not the horizon of the place shown in the figure. Also as was mentioned before in a previous context the time expressed in asus taken by an equatorial arc to rise is equal to the number of minutes in that arc. Since the time taken by the planet to traverse the arc of the diurnal circle namely DG is the same as the time taken by the equatorial arc LM to rise, Bhāskara seeks to find the number of minutes in AB, then compute, the number of Asus taken by LM to rise through which the number of minutes in AC could be computed.

$$AB = AG \sin \widehat{AGB} =$$

$$\frac{AG \times H\sin \widehat{AGB}}{R} = \frac{\beta \times \bar{A}\text{yanavalanajy}\bar{a}}{R} \text{ where } AG = \beta.$$

In the formula given the word Āyanavalana is used for Āyanavalanajyā ie. the Hindu sine of Āyanavalana for brevity. Also, since generally the angle $\widehat{AGB}$ defined as Āyanavalana happens to be small, its Hsine will be almost equal to it. In this sense also the term Valana is used where Valanajyā is to be used.

From AB we pass on to the magnitude of LM. From Hindu spherical Trigonometry

$$LM = \frac{AB \times R}{H\cos AL} = \frac{AB \times R}{H\cos \delta} = \frac{\beta \times \bar{A}\text{yanavalanajy}\bar{a}}{R} \times \frac{}{H\cos \delta} = \frac{\beta \times \bar{A}\text{yanavalanajy}\bar{a}}{H\cos \delta}.$$

(H cos δ is called Dyujyā which is connoted by the term Dyu-guna in the verse, the words Jyā and Guna being synonymous).

$$\text{Thus } LM = \frac{\beta \times \bar{A}\text{yanavalanajy}\bar{a}}{\text{Dyu-guna}}$$

From LM we pass on to the corresponding arc of the Ecliptic namely AC, which does not imply the latitude of the place. So we have to use what are called Niraksha-Udayas of Rāśis ie. the rising times of the Rāśis at Equatorial places. Rule of three is used here to find AC from LM. Locating the particular Rāśi in which the planet is situated, and using the proportion. "If a Rāśi of 30° ie. 1800′ of the Ecliptic rises at an Equatorial place in say x asus, what arc in minutes corresponds to the number of minutes or what is the same the number of Asus in the arc LM ?" We get the magnitude of the arc AC in minutes.

The computation of this arc AC is intended to know the difference between the times of rising of the point A and the point G the former being the point of the ecliptic signifying the position of the planet and the latter being the planet itself. This difference of the times of rising is itself required to know the actual time of rising of the planet by a knowledge of the time of rising of the point which signifies the planet's position on the Ecliptic by its longitude.

In verse 5, Bhāskara gives an alternate and easy method of obtaining the magnitude of AC construing AGC to be roughly a plane triangle which does not involve any appreciable error.

$$AC = AG \tan \widehat{AGC} = \frac{\beta \times H \sin AGC}{H \cos \widehat{AGC}}$$

$$= \frac{\beta \times \bar{A}\text{yanavalanajy}\bar{a}}{\text{Yaṣṭi}}$$ since Yaṣṭi is defined as H cos AGC in verse 3.

Note. The point C is called the Kṛta-Āyana-Dṛk-Karma Sthāna ie. the point of the Ecliptic signifying the planetary position rectified for the Samskāra or correction called Āyana-Dṛk-karma. The longitude of C thus got is called the polar longitude of the planet which is defined as the longitude of the planet as measured on the Ecliptic upto the point of intersection of the planet's declination circle with the Ecliptic. Bhāskara mentions elsewhere,

ध्रुव

ie. in the case of stars which are fixed, the Sphuta-śaras or polar latitudes (given by GC in the above case) given and longitudes rectified for Āyana Dṛk-karma are given ie. polar longitudes. Bhāskara made this as an approximate statement, for, we know, even in the case of stars, though they be fixed, their polar latitudes also do change by the precession of equinoxes be it just a little; whereas their polar longitudes do not change by the same constant quantity as their celestial longitudes.

Verses 6, 7 and 8. Obtain the Carakhandas or ascentional differences of the Sphuta and Asphuta Krāntis. If they be of the same direction their difference is to be taken, if of opposite direction, their sum is to be taken. The result in asus gives the Ākṣa Dṛk-karma correction if the celestial latitude is of appreciable magnitude. If it be not appreciably large, it (the celestial latitude) is to be multiplied by the Ākṣavalana, then divid by H cos ϕ or what is the same, multiplied by the equinoctial shadow and divided by 12. The result is to be multiplied by the radius and divided by H cos δ. Then we have the Ākṣa Dṛk-karma correction in asus. Assuming the planet's position corrected for Āyana Dṛk-karma to be the Sun, obtain the lagna using the asus of the Ākṣa-Dṛk-karma. If the planet has a southern latitude let the lagna be found in the positive direction; otherwise in the negative direction. Then we have the rising lagna of the planet or what is the same the longitude of the rising point of the planet. Again assuming the planet's position increased by 180° to be the Sun, and using the asus of Ākṣa Dṛk-karma, obtain the lagna in the positive direction with respect to a northern latitude of the planet or else in the negative direction. The result gives the longitude of the setting planet or what is called the Asta-lagna.

Comm. (Refer to fig. 110) Let p be the planet whose Graha-sthāna or foot of the latitude is p'. Let q be what is called the Kṛta-Āyana-Dṛk-Karmaka-Sthāna or the planet's position corrected for the correction of Āyana

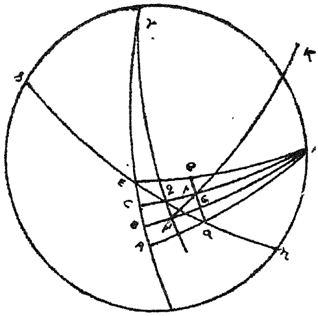


Fig. 110

Dṛk-karma. (This correction is given by the arc $p'q$). The difference of the rising times of the planet p and q expressed in asus is what is sought here. The planet p rose at a and has covered the arc ap of the diurnal circle after rising. So, we have to compute ap and therefrom the corresponding arc of the Equator namely AC. If the planet has no latitude ie. is situated on the ecliptic at p', the arc $p'q$ or Āyana Dṛk-karma vanishes, for, the Āyana Dṛk-karma is no other than the projection of the latitude of the planet on the ecliptic taking P or celestial pole as the vertex of projection. Also if there were no Akṣa ie. if the place be equatorial, p and q rise simultaneously ie. the planet will rise along with the point called Kṛta-Āyana-Dṛk-Karmaka-Sthāna q, mentioned above. In fact the Dṛk-karma corrections Āyana and Akṣa are contemplated to obtain the difference in the rising times of p and p' ie. the planet and its position on the ecliptic. This time is resolved into two parts (i) the difference of the rising times of p' and q and (ii) that of the rising times of q and p. The former is given by the equatorial arc BC which goes by the name Āyana-Dṛk-karma and the latter by the arc AC which goes by the name Akṣa-Dṛk-karma. The algebraic sum of these two corrections gives the difference of the rising time of p and p' ie. the time given by the arc AB. Here AB=AC−BC, In verses

4, 5 we have seen how BC is computed. Here we are seeking to compute AC through the corresponding arc ap of the diurnal circle.

The verse uses the word Sphuta-Krānti, Asphuta-Krānti, and their Cara-khandas. Here in the figure pC is the Sphuta-Krānti and $p'B$ is the Asphuta-Krānti or simply Krānti. Also pp' is called Asphuta-Vikṣēpa and bp' Sphuta-Vikṣēpa. Sphuta-Krānti $= pc = bB = bp' + p'B =$ Sphuta-Vikṣēpa + Asphuta-Krānti. In other words the declination of p, the planet, is called Sphuta-Krānti and that of its longitudinal position on the ecliptic namely p' is called Asphuta-Krānti or simply Krānti. We have obtained Sphuta-Krānti from Krānti by adding to the latter the Sphuta-Vikṣēpa. The method of obtaining Sphuta-Vikṣēpa from the Vikṣēpa, otherwise called Vikṣēpa Sphuta-karama was dealt with in verse 3 above.

Now we have to define the Cara-khandas pertaining to the Sphuta-Krānti pC and the Asphuta-Krānti $p'B$. The former is defined as AE and the latter by CE very approximately. We say very approximately because when p' comes to the horizon, the Cara-khanda will not be exactly EC but a little more or less than EC since $Bp' \neq Cq$. In other words, the arc of the equator between the declination circle of q while rising and the equatorial horizon ie. AE is defined as the Cara-khanda of pC; similarly CE is that of $p'B$. Their difference is $AE - CE = AC$. If we use the modern notation, we have to take their algebraical difference. If that be done, the alternative case of adding the Cara-khandas when the Sphuta and the Asphuta-Krānti are of opposite directions, will be automatically implied. The asus pertaining to this arc AC will be the correction of Ākṣa-Dṛk-karma as mentioned in verse (6) when the latitude of the planet is appreciable. These asus will be equal to the time taken by p the planet to cover the arc ap or what is the same, the time in between the moment of rising of p and that of q, the position of the Kṛta-Āyana-Dṛk-Karmakagraha.

When the latitude is very small, an approximate estimate of this correction is given in asus as

$$\frac{\sin \xi}{\text{H} \cos \phi} \times \frac{\text{R}}{\text{H} \cos \delta} \text{ or } \frac{\beta' \times \text{H} \sin \phi}{\text{H} \cos \phi} \times \frac{\text{R}}{\text{H} \cos \delta}.$$

That this formula is ($\beta'=qp$) only approximate is seen from the fact that H $\sin \xi \neq$ H $\sin \phi$. This second formula can be proved as follows.

Ākṣavalana in asus $= \frac{\beta' \times \text{H} \sin \phi}{\text{H} \cos \phi} \times \frac{\text{R}}{\text{H} \cos \delta}$ where β' is the Sphuta-Vikṣēpa. Taking qpa as a plane triangle

$pa=qp \tan \widehat{pqa}=qp \times \tan \widehat{\text{PE}n}$ approximately

$=$ Sphuta-Vikṣepa $\times \frac{\text{H} \sin \phi}{\text{H} \cos \phi}$. Hence

AC $= \frac{\beta' \times \text{H} \sin \phi}{\text{H} \cos \phi} \times \frac{\text{R}}{\text{H} \cos \delta}$. With respect to the first formula ie.

$$\text{Akṣavalana in asus} = \frac{\beta' \times \text{H} \sin \xi}{\text{H} \cos \phi} \times \frac{\text{R}}{\text{H} \cos \delta},$$

$pa=qp \tan \widehat{pqa} = \frac{\quad}{\text{H} \cos \widehat{pqa}}$

H $\cos \widehat{pqa}$ which is Yaṣti previously defined, H cos ie. H $\cos \phi$, an approximate value is substituted, because the latitude is small. Having obtained the value of pa, it is then reduced to the Equator by multiplying by R and dividing by H $\cos \delta$. The convention with respect to the sign is clear. The idea of Kramalagna and Vilōmalagna may be elucidated as follows. In fig. 110, q, is on the horizon rising whereas p had already arisen. So to obtain the rising time of the planet p, which occured before that of q, Vilōmalagna or an anterior point of time is to be computed, and this is done by subtracting the asus of Akṣa-Dṛk-karma from the rising time of q. The rest follows on these lines of elucidation.

Verse 9. The planet rises at the time known as its Udayalagna (which is to be computed as aforesaid) and it sets at the time known as its Astalagna (which is also to be computed as mentioned before).

Comm. Clear.

Verse. 10. During the night, at a given moment, the computed Udayalagna of the planet is less than the particular lagna of the moment, ie. if the longitude of the Udayalagna is less than that of the current lagna, and also if the Astalagna of the planet computed is greater than that of the then current lagna ie. if the longitude of the Astalagna is greater than that of the current lagna, the planet is visible ie. above the horizon. In the case of the Moon, however, if he is not eclipsed by the rays of the Sun, he may be visible even shortly after Sunrise or a little before Sunset in contradistinction to a planet which could not be seen at all during day time (except perhaps Venus). When a planet is visible, his gnomonic shadow could be computed. (This does not mean that the gnomon casts a shadow of the planet but means that its zenith-distance could be computed according to the methods described in Trīpraśnādhikāra.

Comm. Clear.

Verse 11. To obtain the shadow, the time that has elapsed after the rise of the planet is to be known.

If it be required to find the gnomonic shadow of the planet, then the current lagna and the Udayalagna of the planet *at the moment* are to be computed. The time in between the two lagnas, which will be in Sāvana measure pertaining to the planet gives the time that has elapsed after the rise of the planet.

Comm. According to computation, the difference of times of the current lagna and the Udayalagna as computed taking the present position of the planet will be the time measured along the diurnal path of the planet and as such is in Sānava measure.

Bhāskara mentions here a very subtle point. In the analysis the latitude of the planet is taken into account while finding the Udayalagna. Time measured in this case is called Kṣetrātmika and not Kālātmika for the following reason. Suppose A is the Udayalagna of the planet and B the current lagna both points being on the ecliptic. Let us say that the current lagna is posterior to the Udayalagna (there is no loss of generality in supposing this). By the time of the current lagna, the planet is no more at A but will have moved a little towards B. Let the present position of the planet on the ecliptic be A′. The corresponding arc of AB on the equator gives the sidereal measure of the time that has elapsed after the rise of the planet whereas the corresponding arc of A′B on the equator gives the Sāvana measure of the same time. Here this Sāvana is that pertaining to the planet and not that pertaining to the Sun which is Saura Sāvana. In other words it is the Sāvana pertaining to the particular planet, because the arc AA′ is traversed by the planet in question as per its own velocity.

The stress is here on the word Tat-Kāla-graha, ie. the longitude of the planet at the moment at which the shadow or the zenith-distance of the planet is sought. Taking this as the position of the planet and taking the latitude of the planet also into account compute the Udayalagna. This will be A′ cited above. To obtain the sidereal measure of the same time we have to take A and not A′.

Verse 12. Sāvana measure alone is to be employed while finding the gnomonic shadow ie. while the zenith-distance of the planet is to be computed, because the arc of the diurnal circle of the planet indicates only Sāvana measure. Suppose the Udayalagna falls short of the current lagna, then the Bhōgyakāla ie. the remaining rising time of the Rāśi in which the planet is situated added to the elapsed time of the Rāśi of the current lagna together with the sum of the rising times of the Rāśis in between gives the difference of the Udayalagna of the planet and the current lagna.

Comm. (Ref. fig. 111). Let *p* be the planet's place on the Ecliptic at the moment in question when the lagna is L (or the foot of the latitude of the planet in case it is not on the ecliptic). Let PA be the remainder of the Rāśi in which the planet is situated. Then the time of rising of the arc PA is here termed Bhōgyakāla. Let DL be the arc of the Rāśi in which the current lagna L is situated, which has arisen. The time of rising of this arc DL is called Bhuktakāla. The times of risings of the Rāśis in between namely AB, BC, CD are called Madhyōdayās. The sum of the rising times of PA, AB, BC, CD, DL gives the time in between the rise of P and that of L which is the time that has elapsed after the planet has arisen upto the current Lagna ie. the rising time of L.

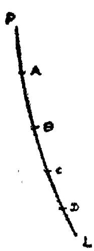


Fig. 111

Verse 13. The method of computing the gnomonic shadow of the planet or what is the same the zenith-distance of the planet, as per the method of computing that of the Sun (extended to the case of the planet) after finding the Sphuṭakrānti (ie. modern declination) of the planet.

The Krānti of the planet or the declination of the foot of the latitude of the planet added to the latitude rectified called Sphuṭasāra, gives what is called Spaṣṭakrānti of the planet and its Hsine is called Spaṣṭa-Krāntijyā. From this H sin δ, H cos δ etc. is to be computed (as mentioned in Trīpras'nādhikāra in the context of finding the zenith-distance of the Sun). From the time that has elapsed after the rise of the planet called the Unnata, the shadow is to be computed as in the case of the Sun's shadow. Having thus computed the shadow or what is the same the zenith-distance of the Moon or that of the stars, the instrument called Nalaka could be pointed to the spot where that celestial body is situated.

Comm. The words Krānti, Spaṣṭakrānti were explained before. Also the method of calculating the zenith-distance from a knowledge of the declination was described before in Trīpras'nādhikāra.

Verses 14, 15. Consideration of horizontal parallax in the case of visibility of the Moon or planets.

H cos z or what is called the Mahās'anku of the Moon or planet is to be reduced by $\frac{1}{15}$th of the respective daily motion gives the visible S'anku when the radius is taken to be 3438. If the radius is taken as 120, $\frac{1}{430}$th of the daily motion is to be subtracted. If H cos z is less than $\frac{1}{15}$th (or $\frac{1}{430}$th) of the daily motion, then the Moon is not visible. This applies to the other planets as well, but as this is a neglible matter, the earlier Āchāryas did not suggest this.

Comm. In the context of the subject of parallax, we mentioned that in the case of the Moon, the horizontal parallax happens to be $\frac{1}{15}$ of the Moon's daily motion approximately. The same is extended to the case of the planets as well. To get the proportion in the case of taking the radius to be 120 only instead of 3438, rule of three is applied as follows. "If the radius be 3438, $\frac{v}{15}$ is parallax, what will it be if the radius be 120?" The answer is

$$\frac{v}{15} \times \frac{120}{3438} = \frac{v}{3438/8} = \frac{v}{430} \text{ very}$$

If H cos z be less than this $\frac{1}{15}$th or $\frac{1}{430}$th of the daily motion, the parallax does not allow the Moon to be seen. Bhāskara specifically talks about the Moon only because, the earlier Āchāryas did not apply the correction of parallax to the case of the planets, because it is not appreciable.

Verse 16. If an operation is neglected because its effect is not appreciable, or is not of much use, or because it is apparent, or if it implies great labour, or again if it implies a lot of exposition that would make the text unduly voluminous, ignoring the necessity of that operation should not be treated as wrong.

Comm. Here Bhāskara upholds the earlier Āchāryas not stipulating parallax in the case of planets, because it is not appreciable.

Here ends Graha-cchāyādhikāra.

GRAHŌDAYĀSTĀDHIKĀRA

Verse 1 and first half of verse 2.

The Udayalagna of a planet is termed Prāk-Dṛk-graha and the Aṣṭalagna is termed the Pas'chima-Dṛk-graha. If the Prāk-Dṛk-graha happens to be less than the current lagna the planet had already risen. If it be greater, the planet is still to rise. Similarly if the Pas'chima-Dṛk-graha is less than the current lagna, the planet had already set; otherwise is yet to set.

Comm. The words Prāk-Dṛk-graha and Pas'chima-Dṛk-graha are coined to indicate the points of intersection of the ecliptic with the eastern and western horizon respectively when the planet is rising or setting, when the planet has latitude. When the planet has no latitude Prāk-Dṛk-graha coincides with the rising planet and the Pas'chima-Dṛk-graha with the setting planet. When the planet has a latitude, the foot of the latitude, which signifies the planet's position on the ecliptic differs from the two Dṛk-grahas. In this case ie. when the planet has a latitude the Prāk-Dṛk-graha's longitude will be less than that of the planet's longitude whereas the Pas'chima-Dṛk-graha's longitude will be greater than that of the planet. If x and y be the differences of the longitudes of the planet and those of the Dṛk-grahas respectively, it is clear from a figure that $\tan x = \frac{\tan \beta}{\tan \theta}$ and $\tan y = \frac{\tan \beta}{\tan \phi}$ where β is the latitude of the planet and θ and ϕ the angles which the ecliptic makes with the horizon at the planet's rising and setting respectively.

Since θ and ϕ are then respectively the zenith-distances of the pole of the ecliptic in each case, and since in the course of half a sidereal day, these zenith-distances could not be equal, x and y cannot be equal.

Second half of verse 2 *and verse* 3. To find the sidereal time in between the given time and the time of the planet's rising.

Find the time that has elapsed after the planet's rise, from a knowledge of the Iṣṭalagna at a given time and Prāk-Dṛk-graha ie. the longitude of the rising point of the ecliptic at the time when the planet rises.

This time will be in Sāvana measure, because we have taken the planet's position at the given moment and not the position of the rising planet. From this time, knowing the daily motion of the planet of the day in question, obtain the arc; that would have been traversed in between the moments. Subtracting this arc from the planet's position at the moment, the planet's position at rising would be obtained approximately; approximately, because as per the procedure enunciated in verse eleven, Grahacchāyādhikāra, the Prāk-Dṛk-graha of the rising planet, and that obtained from the position of the planet at the moment differ. From this approximate time again compute the arc that would have been traversed by the planet during that time. Subtracting this arc which is nearer the truth from the planet's present position, we obtain a more approximate position of the rising planet. Again computing the Prāk-Dṛk-graha from this position, calculate the time from this Prāk-Dṛk-graha and the lagna of the moment. Repeating the process we will obtain the actual sidereal time in between the given time and the actual rising time of the planet. It is sidereal because, we have found the time as per the procedure of verse (11) referred to, where we have used तत्कालखेटोदय and not the actual ,खेटोदय, i.e the rising lagna of the planet obtained from the present position and not that of the rising planet.

This difference between the nature of the times was already dealt with.

The matter of this verse appears to have been unnecessarily complicated by Bhāskara but on careful scrutiny it is not so; for, the problem is to find the time that has elapsed after the rise of the planet upto a given time. Here the data are the given time and the corresponding position of the planet not on the horizon but elsewhere. We could not know immediately the time at which the planet rose. To find it alone, this method of successive approximation has to be used and there is no other go. Had we known the position of the planet while rising, the corresponding Prāk-Dṛk-graha could be found exactly and as this position of the Prāk-Dṛk-graha is a point of the ecliptic and the Iṣṭa-lagna ie. the rising point of the ecliptic at the given moment is also a point on the ecliptic, the time between the moments of rising of these two points of the ecliptic could be found in sidereal measure directly by noting the rising times of the Rāśis in between, without an appeal to the method of successive approximation.

Verse 4. The computation of the times of heliacal rising and setting of a planet in contradistinction to its rising time during a day on account of earth's diurnal rotation.

The times of rising and setting of a planet have been dealt with. Now I shall tell the procedure to be adopted in computing the times of heliacal rising and setting of a planet. If a planet has a daily motion less than that of the Sun, it rises heliacally in the east, and sets in the west; otherwise the reverse.

Comm. Take the example of the superior planets, say that of Jupiter. Since Jupiter moves slower than the Sun, the Sun will have to overtake Jupiter and not vice versa. When it is the Sun that is to overtake, taking a position of Jupiter near the western horizon gradually the planet gets fainter and fainter as the Sun approaches him from west to east. Ultimately one evening the planet ceases to be perceptible within a particular distance of the Sun. This we call heliacal setting of the planet. Having thus set in the west, after a few days the planet rises in the east. This is so because, after the planet's heliacal setting, the Sun gradually approaches the planet from west to east; at a particular moment will have a longitude equal to that of the planet and then gradually gains in longitude over the planet. After a few days, in the eastern horizon, the planet will emerge from the rays of the Sun before Sun-rise; it is not the planet moving west from east but it is the Sun moving from west to east so that the Sun recedes away from the planet towards east. The planet also will be having a motion from west to east but it being slower than that of the Sun, the latter gains over the former in its motion from west to east.

In the case of the inferior planets, say for example, Mercury, the velocity of Mercury is greater than that of the Sun; as such, Mercury will be overtaking the Sun and not vice versa. Thus in the eastern horizon, it is Mercury that enters the rays of the Sun, going from west to east, so that he sets in the east. After a few days of this heliacal setting, Mercury acquires a longitude equal to that of the Sun, and while overtaking the Sun from west to east, he emerges out of the rays of the Sun in the west, which is therefore heliacal rising. But, when Mercury is retrograde, the case is different. Some days after heliacal rising in the west, Mercury attains his maximum elongation and then begins to retrograde. The elongation then gradually decreases, and he will set again in the rays of the Sun in the west itself. A few days thereafter, still continuing retrograde, he emerges out of the Sun's rays in the east, thus rising heliacally. After attaining the maximum elongation in his retrograde motion, his motion will then become direct, so that he again approaches the Sun, going from west to east. Next a little before overtaking the Sun, he sets heliacally in the east, and thereafter, having overtaking the Sun, he rises heliacally in the west.

Here, one point is to be mentioned. The time between Mercury's maximum elongation in the west and again the maximum elongation in the east (which are termed maximum eastern elongation and maximum western elongation with respect to the Sun) will be far less than the time between the maximum elongation in the east (ie. maximum western elongation with respect to the Sun) and that in the west (ie. maximum western elongation with respect to the Sun) in as much as when Mercury is retrograde, and the Sun having always direct motion, the relative velocity will be the sum of the retrograde velocity of Mercury and the direct velocity of the Sun.

In the case of the superior planets, as mentioned above, the superior planets set heliacally in the west and rise heliacally in the east. They will not rise heliacally in the west and will not set heliacally in the east which happens only if either the superior planet has a greater velocity than the Sun or the Sun has a retrograde motion which is never the case.

Verse 5. Speciality with respect to Mercury and Venus.

Mercury and Venus rise heliacally in the west in their direct motion, (attain maximum elongation before they) become retrograde, set heliacally there itself, then rise heliacally in the east continuing to be retrograde, (attain maximum elongation there before they) next become direct and gradually set there (to rise again in the west) as before.

Comm. Explained above.

Verse 6. Kālāms'as or distance in degrees from the Sun within which the planets rise or set heliacally.

The Kālāms'as with respect to the Moon, Mars, Mercury, Jupiter, Venus and Saturn, or the degrees of distance from the Sun within which they rise or set heliacally are 12, 17, 14, 11, 10, 15 respectively. In the case of Mercury and Venus when they are retrograde the Kālāms'as are 12, and 8 respectively.

Comm. The Kālāms'as given above depend upon the luminosity of the respective planets. When Mercury and Venus happen to be retrograde, the Kālāms'as happen to be 2° less in each case because they are then nearest to the earth and as such being most luminous as seen by us will not set heliacally till they are very near the Sun.

Verse 7. To compute the moment when a planet rises or sets heliacally.

If it is to be known when a planet rises or sets heliacally the position of the Prāk-Dṛk-graha or the Pas'chima-Dṛk-graha as the case may be (Prāk-Dṛk-graha in case the rising or setting takes place in the east or the other in the other case) and that of the Sun also are to be computed on a day a little before the day of rising or setting as prognosticated by the S'īghra anomaly. In case the planet rises or sets in the west, to obtain the lagna the position of the Sun is to be increased by 180°.

Comm. Clear.

First half of verse 8. To obtain what are called Iṣṭa-Kālāms'as.

The time between the rising of the planet or of the Dṛk-graha and that of the Sun measured in ghatis multiplied by six gives what are called Iṣṭa-Kālāms'as.

Comm. Having found the approximate position of the Dṛk-graha as mentioned above when the planet is likely to rise or set heliacally, let the time in between the rising of this Dṛk-graha and that of the Sun (if it be the case of setting or rising in the west the position of the Sun is to be increased by 180° because the astalagna directed to be found in verse (1) is the point of intersection of the ecliptic with the *eastern horizon*, which is removed 180° from the setting point of the Sun) be multiplied by six. Since both the Dṛk graha and the Sun's position are points on the Ecliptic and since we have considered the time in between their rising moments, which is measured on the equator, and again since this time is measured in ghatis, the number of ghatis multiplied by six give the degrees, for, each ghati corresponds to six degrees of the equator (sixty ghatis corresponding to one sidereal day). These degrees are said to be Iṣṭa-Kālāms'as, which means that it gives the arc in between the feet of the declination circles of the Dṛk graha and the Sun at the Iṣṭa-kala ie. that particular time considered.

Second half of verse 8 *and verses* (9) *and* (10).

If the number of degrees so found ie. the Iṣṭa-Kālāms'as fall short of or exceed the number of Kālāms'as postulated for the rising or setting of the planet, then the planet's rising has to take place or has already taken place respectively and vice versa in the case of setting. The number of minutes of the difference of the prescribed and Iṣṭa-Kālāms'as multiplied by 1800 and divided by the rising time of the Rās'i expressed in Kalās and again divided by the difference of the daily motions of the planet and the Sun expressed in minutes of arc if the planet is direct in motion or divided by the sum of those daily motions if the planet be retrograde gives the days elapsed after rising or to elapse for the rising to take place. Again, compute the positions of the Dṛk-graha and the Sun for the moment thus obtained and repeat the process till the actual moment is obtained.

Comm. Suppose the prescribed Kālāmśas for rising be x and suppose the Iṣta-Kālāmśas are y such that $y<x$; then for the planet to rise, the arc of the equator in between the feet of the declination circles of the Sun and the Dṛk-graha has to increase for the planet to rise which means that this takes some more time to happen. Similarly if $y>x$, the planet has already risen. The question of the arc decreasing does not arise in this case of rising, because the planet's position in the east is behind that of the Sun, and in the case of a superior planet the Sun has to advance further for the planet to rise ie. the arc has to increase and in the case of a retrograde inferior planet also, the arc will be increasing. In the case of an inferior planet being direct, the arc will be decreasing no doubt, but we have to remember that this is a case of an inferior planet *setting* and not *rising* which we are considering. In the case of setting, in the east, however, of the inferior planet the moment of setting has already elapsed and thus the condition is reverse to that of rising. The case of setting of a superior planet in the east never happens. In the case of setting of a superior planet in the west, if the Iṣta-Kālāmśas be less than the prescribed Kālāmśas, ie. $y<x$, setting must have already taken place, the Sun having approached the planet from behind and already effected heliacal setting. Thus this is also reverse to the condition of rising as stated. In the case of an inferior planet setting in the west if $y>x$, the planet should have set already since the inferior planet is retrograde while setting in the west.

This condition is also reverse to what has been stated in the case of rising. The case of a superior planet rising in the west also never happens, because it is the Sun that overtakes the planet and also the superior planet cannot be retrograde while near the Sun.

The case of rising which has already elapsed the Iṣta-Kālāmśas y will be evidently greater than x ie. $y>x$. To obtain the time by which the rising will take place after the moment in question, ie. the rising of a superior planet which is direct and an inferior planet which is retrograde, we have to take the difference of the prescribed and Iṣta-Kālāmśas and find out the time by rule of three as follows.

(1) Since we have to find the corresponding arc of the ecliptic in minutes of arc, from $(y-x) \times 60'$ the difference of the Kālāmśas of the equator converted into minutes ie. asus we have first to use the proportion. "If by the rising time of the Rāśi t in asus in which the planet is situated we have $1800'$ of the ecliptic, what will we have by $(y-x) \times 60$? The answer is $\frac{(y-x) \times 60 \times 1800}{t}$.

Next we have to find the days when this arc of the ecliptic is covered by the proportion. "If the Sun overtakes the planet by $(u-v)$ minutes of the ecliptic per day, how many days are taken to cover the above arc?" The answer is as stated.

The answer gives in days together with fraction of a day when the planet is likely to rise. We have to repeat the process because the motions of the planet as well as the Sun differ from moment to moment and the time calculated above by rule of three taking into account their motions for the entire day will be only approximate. The computation in the case of setting of a planet may be similarly considered, the setting of a superior planet in the or that of a direct inferior planet in the east.

Verses 11 *and* 12. If the Prāk-Dṛk-graha has a longitude greater than that of the Sun or the Pas'chima-Dṛk-graha has one less than that of the Sun, the sum of the prescribed Kālāms'as and Iṣṭa-Kālāms'as converted into minutes of arc will have to be used to compute the elapsed days or the days after which the respective phenomenon is going to occur.

If the Iṣṭa-Kālāms'as y be greater than the prescribed namely x, the reverse to what has been stated in the second halt of verse (8) and in the first line of verse (9) happens ie. a phenomenon which has elapsed when $y<x$ will happen in the future and vice versa.

Comm. If the Prāk-Dṛk-graha has a longitude greater than that of the Sun, (since the prescribed Kālāms'as are between the planet and the Sun, the planet being behind the Sun, and the Iṣṭa-Kālāms'as are now on the other side of the Sun) the planet has advanced over its position of heliacal setting be it a superior planet or inferior by the sum of the prescribed Kālāms'as and Iṣṭa-Kālāms'as. Hence the days that have elapsed after the heliacal setting have to be calculated with the sum of the two Kālāms'as. If it be also a case of a retrograde inferior Prāk-Dṛk-graha rising, having a longitude greater than that of the Sun, even then the prescribed Kālāms'as are behind the Sun and the Iṣṭa-Kālāms'as ahead of the Sun. So from the position of the planet's heliacal rising, the Sun and the planet have advanced in opposite direction to a distance which is the sum of the prescribed Kālāms'as and Iṣṭa Kālāms'as. Hence computation has to proceed with the sum of the two Kālāms'as to get the time that has elapsed after the planet's heliacal rising. Similar is the argument for the Pas'chima-Dṛk-graha.

We have commented on verses from the latter half of (8) upto (10) where the Iṣṭa-Kālāms'as y happen to be less than x. If $y>x$, or again the Prāk-Dṛk-graha has a longitude greater that of the Sun, the phenomenon of rising or setting which happened when $y<x$, will have to be taken as going to happen and that which was going to happen when $y<x$, must have happened already. It is enough to consider just one case instead of all the four cases since the argument is similar as before. Let us take $y>x$ and it is an inferior planet in the east behind the Sun. According to the latter half verse (8) the rising had to take place y being less than x, whereas, now, y being greater than x, rising had already taken place, in contradistinction to what happens when $y<x$.

Here ends the Udayāstādhikāra.

SṚNGONNATYADHIKĀRA

Verse 1. Either in the last quarter of a lunation, or in the first quarter, on the day when the elevation of the cusps of the crescent Moon is to be determined, then either at the moment of Moon-rise or Moon-set or (for the matter of that during any part of the night) the Hsine of the altitude of the Moon is to be computed by noting the time from the moment of Moon-rise.

Comm. Either in the first quarter or the last quarter of the lunation, ie. when the phase of the Moon according to the definition of phase in modern terms is less than half, the Moon will be a crescent. Also, generally, on the back-ground of the horizon, we notice that one of the cusps is more elevated than the other. This elevation goes by the name Sṛngonnati.

Even in the middle half of the lunation, Bhāskara mentions that Brahmagupta and some others (meaning Sripati whom he closely follows) attempted at finding the elevation (strictly speaking elevation during the second quarter and depression during the third quarter) of the dark horns. Bhāskara does not appreciate this since, nobody would think about this as it does not appeal to the eye at all.

Verse 2. To find the Hsine of the altitude of the Sun.

The Hsine of the altitude of the Sun is to be com-puted, assuming the rising Sun to be in the opposite hemisphere, south or north (ie. if he be originally in the northern, assume him to be in the south) and using the formula given in verse 54 of Tripras'nādhyāya "अथोन्नता-द्नयुताच्चरेण" given the time measured in asus that has elapsed after Sunset.

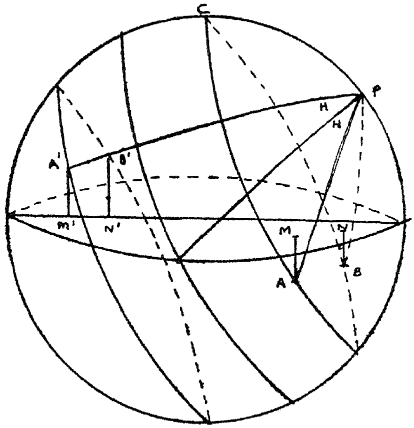


Fig. 112

Comm. During the early part of the night or during the latter part thereof, when the Sun is below the horizon, the Sun will be occupying symmetrical positions with respect to the horizon at times which are equally removed from Sunset and the next Sun-rise. This is clear from the figure 112. Let A and B be two such symmetrical positions where AM and BN are the Hsines of the altitudes. Evidently AM=BN, $\widehat{CPA}=\widehat{CPB}$. Since the rising eastern hour-angle of the Sun equals he setting western hour-angle, and since $\widehat{CPA}=\widehat{CPB}$, the time elapsed after Sunset when the Sun is at B, will be equal to the time before Sun-rise in the position A. Hence by congruence -AM=BN=Hsine of the altitudes in the two symmetric positions. Using the modern formula from the triangle PZS, $\cos z = \sin \phi \sin \delta + \cos \phi \cos \delta \cos h$, when $\widehat{CPA}=\widehat{CPB}=h$ cos z will be the same in the two positions A and B. Putting $z = \overline{90+\theta}$, $\cos z = -\sin \theta$ will be the same ie. H $\sin \theta$ will be the same in the two positions ie. the S'ankus will be the same. In the formula

$\cos z = \sin \phi \sin \delta + \cos \phi \cos \delta \cos h$

Put $z = 90 + \theta$, $h = 90 + H$, $\delta = \delta$ in the positions A, B and $z = 90 - \theta$, $h = 90 - H$. $\delta = -\delta$ in the positions A′, B′

We have then $-\sin \theta = \sin \phi \sin \delta - \cos \phi \cos \delta \sin H$

and $\sin \theta = -\sin \phi \sin \delta + \cos \phi \cos \delta \sin H$

which are identical. This means that the altitude θ below the horizon with $+\delta$ in the positions A, B is computable with $-\delta$ in the positions A′, B′. This accounts for the statement made 'गोलविपर्यये'

The second statement of the latter half of verse (2) says S′ankutala = S′anku $\times \frac{s}{12}$ which we have proved in Tripras′nādhyāya.

Verse (3) *and first half of* (4). To obtain the Bhuja of the Sun.

The S′ankutala of the inverse altitude or the altitude below the horizon, is north (in contradistinction to what it is above the horizon). The sum or difference of the Agrā and S′ankutala according as they are of the same direction or of opposite directions, is the Bhuja. The sum or difference of the Bhujas of the Sun and Moon, according as they are of opposite or the same directions is what is called the Spaṣṭa Bhuja whose direction is to be construed as that of the Moon. If the Bhuja of the Moon falls short of that of the Sun, then the direction of the Spaṣṭa Bhuja is that opposite to that of the Moon.

Comm. In fig. 113, we have shown five different positions of the Sun S_1 to S_5 the feet of the S′ankus being B_1 to B. From these feet of the S′ankus draw perpendiculars on the Udayāstasutras as well as on the East-west line their points of intersection being respectively A_1 to A_5 and C_1 to C_5. The perpendiculars from the feet of the S′ankus on the East-west line go by the name Bhujas, the perpendi-

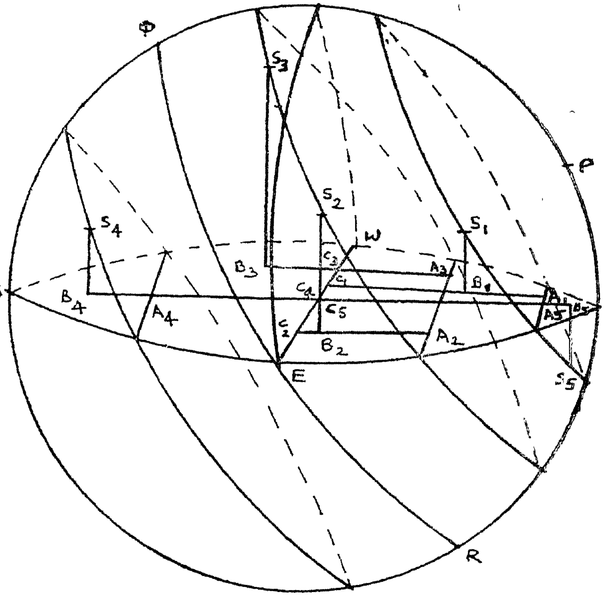
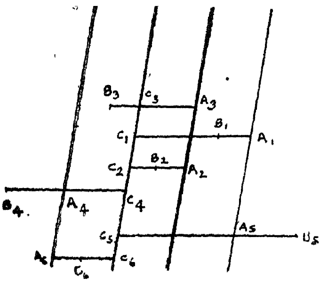


Fig. 113

cular distances between the Udayāstasutras and the East-west line are called Agrās and the perpendiculars from the feet of the S'ankus on the Udayāstasutras are called S'ankutalas which were all defined in the course of the Tripras'nādhyāya. We have from the spherical triangle PZS, $\sin \delta = \sin \phi \cos z + \cos \phi \sin z \sin a$ so that $\frac{\sin \delta}{\cos \phi} = \tan \phi \cos z + \sin z \sin a$ which was shown as $A = S + B$ in the Tripras'nādhyāya ie. Agrā = S'ankutala + Bhuja. Changing δ into $-\delta$ and a into $-a$ we have different formulae, the standard form being $A = S + B$. We don't propose to change ϕ into $-\phi$, because in India this case does not arise. Thus our latitude being north, the S'ankutalas defined above as the distances from the feet of the S'ankus from the Udayāstasutras are always deemed as south. It need not be reiterated that the Udayāstasutras are the straight lines joining the rising and setting points of the celestial body. As such, these Udayāstasutras are all parallel to the East-west line. In fig. 114, in the horizontal



Fig, 114

plane containing the points A′s, B′s and C′s in the respective cases, according to the Hindu convention, we talk of Agrā and Bhuja as being Uttara Agrā, Dakṣināgrā, Uttara Bhuja and Dakṣina Bhuja. S′ankutalam is always taken to be south except in the fifth case shown in figs. 113 and 114 when the altitude happens to be below the horizon. In this case the S′ankutala is spoken as north. Thus the general formula $A = S + B$ assumes the following various forms, in the respective cases.

(1) Uttarāgrā—Dakṣina S′ankutala = Uttara Bhuja. This corresponds to A, B, C of fig. 114 where AC, is Uttarāgrā, BA, Dakshina S′ankutala and BC, Uttara Bhuja.

(2) In the case of $A_3 B_3 C_3$, S′ankutala—Agrā = Dakṣina Bhuja since $B_3 A_3 - C_3 A_3 = B_3 C_3$. This occurs after the Sun's diurnal path has crossed the prime-vertical and the Sun has a position on the south of the prime-vertical, having a northern declination. In the case of $A_4 B_4 C_4$, $B_4 C_4 = B_4 A_4 + A_4 C_4$ ie. Dakṣina Bhuja = Dakṣina S′ankutala + Dakṣināgrā. In the fifth case when the Sun's altitude is below the horizon, $B_5 C_5 = B_5 A_5 + C_5 A_5$ ie. Uttara Bhuja = Uttara S′ankutala + Uttarāgrā. In the sixth case when the Sun has a southern declination and an altitude below the horizon, $B_6 C_6 = A_6 C_6 + A_6 B_6$ ie. Dakṣina Bhuja = Dakṣināgrā minus Uttara S′ankutala. In the case of $A_2 B_2 C_2$, Uttarāgrā =

Dakṣiṇa S'aṅkutala + Uttara Bhuja, since $A_2 C_2 = B_2 A_2 + B_2 C_2$. In the absence of a unifying convention, all these formulae are loose and apt to create confusion. So we shall unify all these formulae into the standard form A=S+B which holds good universally, if we have the conventions.

(1) Agrā shall be deemed positive if it be north and negative if it be south;

(2) Similarly with respect to the Bhuja. But, with respect to the S'aṅkutala, we shall deem it positive if it be south and negative if north. These conventions comprehend all the cases and unify them into the standard form A=S+B. The corresponding conventions with respect to δ, a are that $+\delta$ corresponds to Uttarāgrā and $+a$ corresponds to Uttara Bhuja.

Having the above conventions, and finding the Bhujas of the Moon above the horizon and the Sun below the horizon, the Spaṣṭa Bhuja required in finding the S'ṛṅgonnati in the present chapter, is defined as the difference of the Bhujas of the Sun and the Moon and the sum thereof according as they are of the same direction north or south or of different directions. Also this Spaṣṭa Bhuja is by convention said to have the same direction as that of the Moon. If, however, the Bhuja of the Moon falls short of that of the Sun, then the Spaṣṭa Bhuja is said to have the direction opposite to that of the Moon.

Verse (4). Definition of Koṭi.

I deem that the Koṭi should be taken as the sum of the S'aṅkus of the Sun and the Moon, the one being below the horizon and the other above respectively.

Comm. We have defined above the Spaṣṭa Bhuja as the north-south distance between the feet of the S'aṅkus of the Sun and the Moon. Herein Bhāskara defines the Koti as the sum of the S'ankus of the Sun and the Moon which is the vertical distance between the points on the armillary sphere which represent the Sun and the Moon.

Bhāskara says 'I deem' to signify that he differs from Brahmagupta in this, who defines the chord joining the Sun and Moon on the armillary sphere which is equal to 2 Hsine of the SM on the sphere as the Karṇa. Brahmagupta defines the Bhuja and Karṇa from which he deduces the Koti as $\sqrt{\text{Karṇa}^2 - \text{Bhuja}^2}$. Bhāskara argues that since the Karṇa defined by Brahmagupta is not in the vertical plane, since the Sun and the Moon are not in the same vertical plane, so, the Koti defined by him throught he Karṇa will not be in the vertical plane. The Karṇa defined by Bhāskara is not on the other hand the chord joining the Sun and the Moon but lies in the vertical plane in which the perpendicular from the Moon's position on the armillary sphere, on the horizontal plane eontaining the Sun's position on that sphere. Also the Bhuja defined both by Brahmagupta and Bhāskara is the projection on the north-south line of the join of the Sun's position on the sphere and the foot of the perpendicular from the Moon's position on the horizontal plane through the Sun's position and it is not actually the above join. Bhāskara is evidently guided by the right angled triangle SMN which is not strictly a spherical triangle as per its modern definition as the arc SN is not that of a great circle. This figure guided him to take the Bhuja horizontal and the Koti vertical so that his Karṇa is also in a vertical plane. Bhāskara's

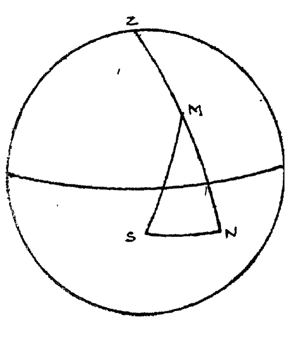


Fig. 115

Karṇa therefore has nothing to do with the arc SM but only has the virtue of being in a vertical plane. Sripati adopted Brahmagupta's method. Kamalākara neither follows Brahmagupta nor Bhāskara but follows a method of his own. In fact the methods adopted by these ancient Hindu astronomers are not mathematically correct because the question of determining the cusps as well as phase of the Moon is concerned with the actual positions of the Sun, Moon and the earth in space and not as seen on the sphere. Hence M. M. Sudhakara Dwivedi has written a small book by the name Vāstava-Sṛngonnati following modern methods. The modern method which gives the truth of the matter is depicted in standar modern texts.

Verse 5. The hypotenuse or Karṇa is the square-root of the sum of the squares of the Bhuja and Koti. The Bhuja multiplied by 6 and divided by the Karṇa gives what is known as the Dik-valana of the Moon. The direction of the Valana has the same direction as the Spaṣṭa Bhuja defined before.

Comm. The idea is that taking the radius of the Moon's disc to be six angulas or units, representing the Karṇa, the magnitude of the Bhuja on the same scale gives a measure of what is defined as Valana. Thus Valana $= \frac{6B}{K}$ where B and K stand for the Bhuja and Karṇa defined before. The idea of this Valana will be clarified in the ensuing verses.

Verse 6. The Hsine of the elongation of the Moon is to be multiplied by the radius vector of the Moon measured in Yojanas, and divided by the radius vector of the Sun, also measured in Yojanas; the arc of the Hsine so obtained is to be added to the longitude of the Moon in the bright half of the lunation and is to be subtracted from the same in the dark half.

Comm. This is a correction to be made in the longitude of the Moon to depict graphically the phase of the Moon. Bhāskara says that many of the prior astronomers took that the phase of the Moon was in direct proportion to the elongation of the Moon. Taking the radius of the Moon's disc to be six angulas or units, and assuming that when the elongation is 180°, the entire disc being illuminated, it was thought that 12 units of the diameter correspond to 180° of elongation so that the S'ukla of the disc measured by the central width of the illuminated disc increases at the rate of 1 unit for 15° of elongation. (The word S'ukla may be taken to correspond to the modern word phase. S'ukla is expressed in angulas, taking the diameter of the disc to be 12 angulas. The measure of the portion of the diameter of the Moon's disc perpendicular to the diameter which is the join of the extremities of the cusps, covered by the illuminated part of the disc, (which may be defined as the maximum width of the illuminated part of the disc) measured in angulas is said to be the S'ukla. The word 'phase' is used to signify the ratio of the above width to the diameter. S'ukla is expressed in angulas, whereas phase is expressed as a ratio. The S'ukla is equal to twelve times phase). Bhāskara rightly argues that this method of measuring the S'ukla is approximate because he says that six angulas of S'ukla is had when the elongation is not 90°, but only 85°–45′. This may be substantiated as follows from fig. 116. Let E, M, S stand for the earth, Moon and the Sun. Let $\widehat{EMS}=90°$ so that it is a moment of dichotomy ie. the moment when the phase is half and the S'ukla 6 angulas. Let θ be then the elongation of the Moon, so that $\cos\theta = \frac{EM}{ES}$. Taking the average values given for EM and ES by Bhāskara,

$$\frac{51566}{689377} \quad \frac{19}{254} \text{ (obtaining a convergent)} = .0748.$$

From tables, we find $\theta = 85°\text{–}43'$ which Bhāskara takes to be $85°\text{–}45'$.

Fig. 116

In the wake of this, Bhāskara tries to make amends in the approximate formula prescribed to obtain the S'ukla. He prescribes addition of $4°\text{–}15'$ to the longitude of the Moon in the bright half, and subtraction in the dark. In between the moment of conjunction and the moment of dichotomy, he derives the following formula (vide fig. 117).

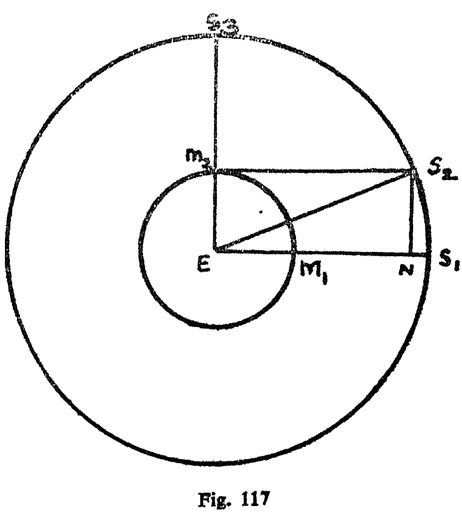


Fig. 117

The deficiency of $4°\text{–}15'$ is had in the form of the angle θ_1. When the Sun is at S_1 the Moon being at M_1, it is the moment of conjunction. When the Sun has moved from the point S_1 to S_3, $S_1 \hat{E} S_3$ being 90°, there is a deficiency of magnitude $S_1 E S_2$. In other words, when $S_2 N$ has assumed the position $S_3 E$ there is a deficiency of 4°–15′ ie. for an increase of 90° of elongation, there is a deficiency of 4°–15′ in the longitude of the Moon. Hence, for the Hsine to become the radius, there corresponds a portion $E M_2$ or $S_2 N$, which is the Hsine of 4°–15′, so that the following rule of three is adopted. 'If the Sun's distance $E S_3$ corresponds to the radius, what does $E M_2$ the distance of the Moon correspond to?" The result is $\frac{m}{s} \times R$, m and s being the respective distances. Then the following proportion is used "If by H sin ξ equal to R, ξ being the Moon's elongation, we have mR/s, what shall we have for an arbitrary H sin ξ?" Thus the answer is

$$\text{H sin}\,\xi \times \frac{mRs}{R} = \text{H sin}\,\xi\,\frac{m}{s}. \qquad \text{H sin}^{-1}\left(\frac{m}{s} \times \text{H sin}\,\xi\right)$$

where m and s are the distances of the Moon and the Sun, is to be added to the longitude of the Moon or to be subtracted as the case may be, to have the rectified longitude of ths Moon from which the phase is to be calculated according to Bhāskara.

Here we are to offer the following remarks. No Bhāskara was correct in estimating the moment of dichotomy to be that when ξ the Moon's elongation is not 90° but 85°–45. But the amended formula is not the correct mathematical form. The modern formula to find the phase is $\frac{1+\cos \text{EMS}}{2}$ aud since from fig. 116, SM is nearly equal to SE, so EMS is very nearly equal to

180–MES, so that $\frac{1+\cos EMS}{2} = \frac{1-\cos MES}{2}$ which may be taken to be an approximate truth. This formula was indeed given by Lallācārya in the following verse, long before Bhāskara "रविशीतकरान्तरांशजीवा विपरीता शशिखण्डताडिता च, विहृता त्रिभजीवया सितं स्यात् शशलक्ष्माङ्गवदङ्गुलानि तस्मिन्" verse 12 (Candra S'ṛngonnatyadhikāra). Here विपरीता रविशीतकरान्तरांशजीवा mean Hvers ξ = R—H cos ξ. शशिखण्डताडिता = multiplied by the radius of the disc of the Moon. विहृता त्रिभजीवया = divided by R. Thus the formula given by Lallācārya amounts to $r\,\frac{(R-H\cos\xi)}{R} = \frac{d}{2}(1-\cos\xi)$

where r gives the angulas in the radius of the Moon's disc and d the diameter, and ξ = elongation of the Moon. Since the definition of phase in modern terms is a ratio, namely the ratio of the maximum width of the crescent to the diameter, phase $\times\, d$ = S'ukla of the Hindu astronomers

$$\therefore \quad \frac{1-\cos\text{(elongation of the Moon)}}{2} \times d = \text{Sukla}$$

$= r\,(1-\cos MES)$ as given by Lallācārya. So Lallācārya's formula is quite correct. We shall trace Lallācārya's steps in obtaining such an intricate correct formula which was overlooked by Bhāskara. (Refer fig. 118) Let E be the earth, S the Sun and M_1, M_2, M_3 etc. the positions of the Moon as the elongation gradually increases. No doubt the S'ukla increases with elongation as known to all Hindu astronomers. But, the question is, does it increase with the sine or versine? We know both the sine and versine increase with the angle. Lallācārya noticed that Sm_1, Sm_2, Sm_3 as the Moon occupied positions M_1, M_2, M_3 etc., are the Hindu versines which are increasing. So he postulated that the S'ukla increases with the Hindu versine. His formulation was thus correct. But why Bhāskara overlooked this correct formula was traceable to his getting prejudiced

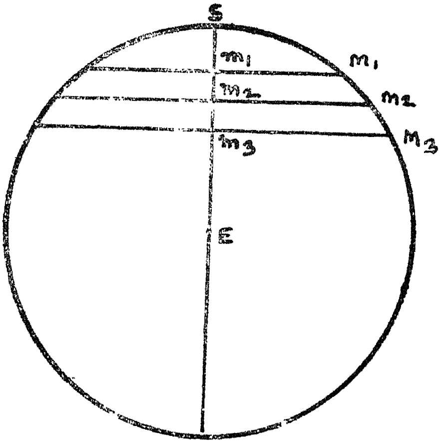


Fig. 118

against Lallācārya's some other formulae which used the Hversine where he ought to have used the Hsine. For example in the case of the Valana Lallācārya's mistake is traceable to his confusion as to whether he was to choose the Hsine or Hversine when both increase as the angle increases.

Verse 7. Graphical depiction of the cusps.

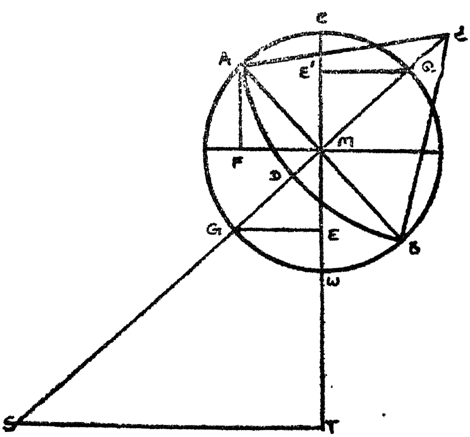


Fig. 119

Let in fig. 119, ST be the Bhuja, MT the Koti and SM the Karṇa formerly defined. As per verse (5) $\frac{B}{}$ = Valana where B = ST, K = SM, and 6 = MG so that $GE = \frac{6B}{K}$ = Valana, which is in practice drawn as E′G′ from the east point *e*, in the form of a Hsine. CG goes by the name Valanasūtra. Compute the S′ukla in angulas after making the prescribed correction in the longitude of the Moon as stated by Bhāskara and dividing the elongation of the Moon, thereafter obtained, by 15. Mark off the S′ukla in angulas along the Valanasūtra from G. Suppose GD is the S′ukla. Draw the diameter AB perpendicular to the Valanasūtra passing through M. Draw the circle circumscribing D, A, B. Its centre lies evidently on the Valanasūtra, say C. The circle drawn is called Parilekha Vṛtta, its radius CA is called Parilekhasūtra and the point C Parilekha Vṛtta Madhya. In the triangle CAM, which is right-angled CA is the Karṇa, AM the Bhuja and MC the Koti. CD is equal to the Karṇa CA so that MD = CD − CM = Karṇa − Koti = $K-k$ (say). We have $AM^2 = CA^2 - CM^2 = K^2 - k^2 = 36 = B^2$; Hence $K + k = \frac{B^2}{K-k}$.

Here $K-k$ = MD is known because GD the S′ukla is known. So $\frac{B^2}{K-k} = K+k$ is known. Thus knowing $K-k$ and $K+k$, by using what is called Samkramagaṇita ie. by adding $K+k$ and $K-k$, we have K and by subtracting we have k. Here the Koti CM is called Vibhā and the Karṇa CD the Swabhā. The Parilekhasūtra or the radius of the Parilekha Vṛtta being the Karṇa, the Swabhā is thus the Parīlekhasūtra.

In the wake of this exposition, the translation of verse (7) runs as follows. "Let the compliment of the elongation (corrected as directed in verse (6)) divided by 15 be the denominator; let the numerator be 36; take the result after division and put it in two places; with this and the complement of the elongation divided by 15, using Samkramagaṇita, we have successively Vibhā and Swabhā".

Comm. Here, the meaning of taking the complement of the elongation and dividing by 15 is to obtain the magnitude of MD directly; for, $\frac{90-e}{15} = 6 - e/15 =$ $MG - e/15 = MG - GD = MD$ where e is the elongation and $e/15$ is the S'ukla. In other words, finding the S'ukla by dividing the elongation by 15 and subtracting it from the radius to get MD is the same as taking the complement of the elongation and dividing it by 15 to obtain MD.

Bhāskara quotes in the course of the commentary, the verse from his Leelāvatī, namely "भुजाद्वर्गितात् कोटिकर्णान्तरान्तम् द्विधा कोटिकर्णान्तरेणोनयुक्तम्। तदर्धे क्रमात्कोटिकर्णौ भवेताम्, इदं धीमताऽऽवेद्य सर्वत्र योज्यम्" which means $\frac{B^2}{K-k} = K+k$ since $K^2 - k^2 = B^2$. This situation arises when the Bhuja B and the difference of Koti and Karṇa ie. $K-k$ are given and it is required to find K and k separately.

Verses (8) and (9). Draw a circle with radius 6 angulas to represent the disc of the Moon. Mark the disc with the Cardinal points signifying the east, west, north and south. Compute the Valana as directed in verse (5) (which represents E′ G′ in fig. 119) and mark it off as a Hsine from the west point w in the last quarter and from the east point e in the first quarter. From the centre M of the Moon's disc, mark off Vibhā MC (computed as directed) along the join of MG′. With centre C and radius Swabhā (already computed as directed) draw a circle. The spherical sector of the Moon's globe thus demarked by the arc of the circle ADB namely AGBD is found to have the elevated cusp in the opposite direction in which the Valana has been marked.

Comm. The directions are clear in the wake of the previous commentary. In marking off the Valana, Bhāskara says 'from the west in the last quarter and from the east in the first quarter'. In the first quarter, the Moon will be visible immediately after Sunset in the western sky and the illuminated part of the Moon will be towards the western side of the disc in which direction the Sun is situated. The eastern point and the western point of the disc are easily discernible on the background of the horizon. Drawing first the diameter 'WE' of the disc along the vertical circle in case the Moon is due west or else along a small circle parallel to the prime-vertical called Upa-Vṛtta, we have the east point of the disc namely *e* vertically above the disc in the first quarter. (The figure 119 is shown for the first quarter). Then we are directed to mark off the Valana E′ G′ in the form of a Hsine'. Since the Valana is already computed as directed in verse (5), to mark off this Valana we have to lay the scale perpendicular to WE. Such a point G′ on the circumference of the disc is to be marked from which the Hsine equals the Valana computed. It is to be noted here that it is not marking G′ E′ from G′, for, G′ is not known, nor E′ for the matter of that, but locating G′ knowing the magnitude of the Valana. Though we happened to mention before that GE is the Valana, in the light of what Bhāskara mentions in verse (8) we have to understand that in the first quarter, the practice is to mark off Valana from the point *e* marked on the disc before hand. Though it does not matter when we take EG to be the Valana E′ G′, we have to notice the practice followed. The direction given in the course of the commentary "केन्द्राङ्कलनोपरिवृत्तात् बहिरपि खटिकया सूत्रमुच्छ्राद्यम्" meaning thereby that from the point M, we have to mark off along MG′ where G′ is the point pertaining to Valana, a line with the help of a chalk and a fine thread, namely MC. This practice is still in vogue adopted by masons to draw straight lines.

It is interesting to note Bhāskara waxing poetic in the course of the commentary under these verses describing how the illumination of the Moon's disc by the rays of the Sun is effected. The description is quite apt, interesting and scientific reminding us of Varāhamihirācārya's description in the verse 'सलिलमये शशिनि etc.' in Bṛhat-Saṁhitā. Of course, following Varāhamihira Bhāskara also takes the Moon's globe to contain water within its bosom which Modern Science has yet to confirm.

Verses 10, 11, 12. The Koti and Karṇa defined by Brahmagupta, do not lead us to accordance between computation and observation to locate the cusps. I request good mathematicians to verify this carefully. In a place where the latitude is $(90-\omega) = (90-24) = 66°$, when the ecliptic coincides with the horizon, and when the Sun is in the beginning of Mēṣa which is then rising in the east, and the Moon in the beginning of Makara, then the Moon is dichotomized by the meridian and the illuminated part of the Moon's disc is towards the east. This does not hold good according to Brahmagupta's definition of Koti. because then the Bhuja as well as Koti according to his definition is equal to R. When the Bhuja is zero, the cusps will be horizontal, and when the Koti is zero, they will be vertical. Brahmagupta's Bhuja and Koti both being equal to R, the cusps therefore cannot be vertical which is against truth as stated above. Why should I make this statement? May my homage be to those great.

Comm. When the latitude of a place be $90-\omega$, and when the ecliptic coincides with the horizon, it is clear that in whatever positions be the Sun and the Moon (assuming that the Moon is also approximately on the horizon) the cusps of the Moon are always vertical, there being Bhuja only and no Koti. But according to Brahmagupta's definition of Karṇa, it will be in this particular case. $\sqrt{R^2 + R^2} = R\sqrt{2}$ so that the Koti will be $\sqrt{2R^2 - R^2} = \sqrt{R^2} = R$, where the Bhuja also is R (projection of the line joining the Sun and the Moon on the north-south line. Though in the verse (11) above one particular position of the Sun and one of the Moon, are contemplated, the same argument holds good, says Bhāskara in the course of the commentary, whatever positions are occupied by the Sun and the Moon on the ecliptic. In this case there is only Bhuja existing and no Koti, so that the cusps will be vertical. But in all these cases, there is Koti according to Brahmagupta's definition, so that the cusps will not be vertical according to him, which is clearly against truth.

Here we have to note that in the example cited by Bhāskara the Bhuja equal to R is along the north-south direction, and even though the Koti according to Bhāskarajs definition is conceived to be vertical, the Karṇa according to Brahmagupta's conception being ES where E and S are the east and south points, his Koti will be horizontal coinciding with OE, O being the centre of the horizontal ecliptic. Further according to Brahmagupta, the join of the cusps will be perpendicular to the Karṇa which does not therefore go against truth. Similarly in all the cases wherever be the Sun and the Moon on the horizon, Brahmagupta's Karṇa being a line joining the centres of the discs of the Sun and the Moon, it will be a horizontal line and so the cusps could be vertical. It is not clear whether or not Bhāskara recognized this namely that the Karṇa and Koti of Brahmagupta in the cases cited above are all horizontal lines so that the cusps could be vertical even according to Brahmagupta. That is why he pays homage to Brahmagupta in the last line of (12). Probably Bhāskara expected that Brahmagupta's Koti also should have been vertical as his. The fact that he recognized that Brahmagupta's Koti will not be vertical but inclined, is justified here since the Kotis in all the cases cited are all horizontal lines.

In fact, as we stated before, even Bhāskara's analysis is not sound, for which reason, M. M. Sudhākara Dwivedi, wrote the booklet called Vāstava Sṛngonnati based on modern lines. As this topic could be found in books of modern astronomy, we need not go into the treatment of Sudhākara Dwivedi here.

Here ends the Sṛngonnatyadhikāra

GRAHAYUTYADHIKĀRA

Verse 1. The mean diameters of Mars, Mercury, Jupiter, Venus and Saturn are respectively 4′–45″, 6′–15″, ′–20″ 9′. 5

Comm. Bhāskara gives us a method under verse (5) of Chandragrahaṇādhikāra, as to how the angular diameters of celestial bodies were being measured with an instrument that we may call 'protractor', describing it as follows. "यस्मिन् दिने अर्कस्य मध्यतुल्या स्फुटा गतिः स्यात् तस्मिन् दिने उदयकाले चक्रकलाव्यासार्धमितेन यष्टिद्वितयेन मूलमिलितेन तत्रस्थदृष्ट्या तदग्राभ्यां बिम्बप्रान्तौ विध्येत् "या यष्ट्यग्रयोरन्तरकलाः ता रविबिम्बकला भवन्ति मध्यमाः" ie. On the day on which the true motion of the Sun equals the mean, *in the morning*, observe with an instrument having two equal rods jointed at one end (and the other ends being connected by a flexible protractor marked with minutes and seconds of arc) placing the eye at the joint of the rods, and the two rods pointing to the extremities of a diameter of the disc. The magnitude of the disc is then read on the protractor, which gives the mean diameter".

This method is alright so far as it goes. The measuring being done at the time of morning and that too with the naked eye might have been responsible for the exaggerated magnitudes of the diameters of the discs of the planets, as compared with their modern values. This kind of exaggerate estimate is due what is called the phenomenon of irradiation which increases the apparent size of a brilliant body when seen at some distance. Even Tycho Brahe, an accurate and brilliant astronomer prior to the invention of the telescope gave estimates of these angular diameters which are nearly the same as remarked by Burgess in his translation of Sūryasiddhānta under verses 13, 14 ch. VII.

Verse 2. These estimates being multiplied by the difference of the radius and S'īghrakarṇa and divided by thrice the S'īghra-antyaphlajyā are to be added or subtracted from the mean values above given according as the S'īghrakarṇa is less or greater than the radius to give the rectified values. Three minutes of arc are to be construed as one angula in this respect.

Comm. When the S'īghrakarṇa equals the radius we know that the planet is situated at the mean distance. The word planet here stands, of course, for the Mandaspaṣṭagraha which may be roughly taken to be the mean planet, the equation of centre being small. If the S'īghrakarṇa falls short of the radius, then evidently the planet is nearer the earth than the mean position so that disc of the planet appears to be bigger. Otherwise, the planet is further and its disc appears to be smaller. It was noted that approximately there was an increase or decrease $\frac{1}{3}$ of the magnitude of the disc by the decrease or increase of the S'īghrakarṇa by the antyaphalajyā. Hence, in between the two positions, rule of three is used to obtain the magnitudes as follows. "If by a difference of the S'īghrakarṇa and radius equal to the antyaphalajyā, there is a difference of $\frac{1}{3}$ of the actual magnitude, what would it be for an arbitrary difference ?" The answer is $d \times \frac{1}{3} \times \frac{1}{a} = \frac{d}{3a}$ where d = S'īghrakarṇa or radius and a the antyaphalajyā. This difference is to be added or subtracted to the mean value, as the case may be.

Verse 3 *and first half of* 4. To obtain the time of the conjunction of two planets, compute the difference of the longitudes of two planets, and divide by the difference of their daily motions. If one of the planets be retrograde, divide by the sum of the daily motions. The result gives the number of days approximately after the moment of conjunction if the slower planet has a longitude falling short of that of the quicker. If one of the planets be retrograde, and if its longitude be the lesser then also the conjunction was past by the number of days computed. In the other cases the conjunction is to take place after the number of days computed. If, however, both the planets be retrograde, then if the slower of them has a longitude less than that of the quicker, then the conjunction is ahead, otherwise past by the number of days.

Comm. Clear. In the case of one or both the planets being retrograde, the word 'slower planet' means that planet whose retrograde motion is slower and not the planet whose mean motion is slower.

Latter half of verse 4 and verse 5. To rectify the moment of conjunction.

Having computed the approximate time of conjunction, obtain the true motions of the planets pertaining to that day, rectify them for Āyana-Dṛk-Karma and following the process indicated in verse (3) above, again compute the moment of conjunction. (This will be a good approximation). This conjunction will be one on the polar latitudinal circle. If Āyana-Dṛk-Karma be not done, then the conjunction will be on the circle of celestial latitude.

Comm. Bhāskara says that the conjunction on the circle of polar latitude is preferred because this could be observed as there is a star at the celestial pole and the movable circle of polar latitude could be moved into a position in which the two planets could be seen situated thereupon. The method of successive approximation is self-explanatory. Bhāskara, however, adds that when the conjunction is on the circle of celestial latitude, the planets will be seen to be closer.

Verse 6 and first half of verse 7. To obtain the north-south celestial latitudinal distance between two planets.

Having computed the number of days by which the celestial latitudinal conjunction was past or is going to take place, let the common celestial longitude of the two planets be obtained by the method of successive approximation for the moment of celestial latitudinal conjunction. Let the celestial latitudes of the two planets be rectified for parallax in latitude as in the case of solar eclipse. The difference of these celestial latitudes in case they are of the same direction or the sum if, of opposite direction, gives the north-south distance of the planets with respect to the ecliptic. Having known the directions of the celestial latitudes with respect to the ecliptic, if the two celestial latitudes happen to be both south or both north, then the planet with lesser celestial latitude is said to be in the opposite direction with respect to the other; that is, suppose both have northern celestial latitudes and suppose p_1 has a smaller northern celestial latitude than p_2, then p_1 is said to be south of p_2. Similar is the case if both the celestial latitudes happen to be south.

Comm. Here Bhāskara does not specify whether he is talking of conjunction on a polar latitudinal circle or on a celestial latitudiaal circle, though the previous procedure indicated by him to obtain the moment of conjunction gives preference to polar latitudinal conjunction which is more easily observable. But, here, as he prescribes rectification of the latitudes for parallax in celestial latitude, we have to construe that he is speaking of conjunction with respect to celestial longitudes alone, because, in the context of parallax, no method was indicated by him for parallzx in polar latitude.

Latter half of verse 7 *and verses* 8 *and* 9. Case of occultation.

If the north-south celestial latitudinal distance happens to be less than the sum of the angular radii of the two planets, an occultation occurs (what we say 'eclipse' with respect to the Sun and the Moon, holds good with respect to occultation). In this case of occultation, we have to rectify the time of conjunction with respect to parallax in latitude also. For obtaining this parallax in longitude, let the planet which is nearer the earth be taken as the Moon and the other the Sun. But to obtain the longitude of the Vithrībha, which is necessary to compute parallaxes in longitude and latitude, the lagna of the moment of conjunction is to be computed from the position of the Sun and not that of the planet assumed to be the Sun as directed above. Having obtained the parallax in longitude, the computed moment of conjunction is to be rectified for parallax in longitude, (if necessary by the method of successive approximation) to obtain the actual moment of apparent conjunction ie. occultation here. This procedure is worth-adopting only when the occultation in question takes place above the horizon and is observable. The north-south celestial latitudinal distance in this case of occultation corresponds to the celestial latitude of the Moon in the case of a solar eclipse. The direction of this celestial latitude is to be construed as that of the direction in which the planet near the earth is situated with respect to the other. If the planet which is nearer the earth happens to have a motion lesser than that of the other, or be retrograde, then the planet which is situated at a greater distance from the earth will be over taking the other so that the higher planet gets occulted in the eastern direction of its disc. Thus the first contact is to be known to be in the east and the last contact would be in the west; (If otherwise, the other way).

Comm. Self-explanatory.

Here ends the Grahayutyadhikāra.

Bhagrahayuti (Conjunction of a planet with respect to a star)

The longitudes of the stars (professed to be polar).

The polar longitudes of the stars from Aswini including Abhijit are as follows.

	R d m		*R d m*
As'winī	0- 8- 0	Swātī	6-19- 0
Bharaṇī	0-20- 0	Vis'ākhā	7- 2- 5
Kritticā	1- 7-18	Anūrādhā	7-14- 5
Rohiṇī	1-19-18	Jyeṣthā	7-19- 5
Mṛgas'īrṣa	2- 3- 0	Mulā	8- 1- 0
Ārdrā	2- 7- 0	Purvāṣādhā	8-14- 0
Punarvasū	3- 3- 0	Uttarāṣādhā	8-20- 0
Puṣyamī	3-16- 0	Abhijit	8-25- 0
Asreṣā	3-18- 0	Sravaṇam	9- 8- 0
Makhā	4- 9- 0	Dhaniṣthā	9-20- 0
Purvāphalgunī	4-27- 0	S'atabhiṣak	10-20- 0
Uttarāphalgunī	5- 5- 0	Purvābhādrā	10-26- 0
Hasta	5-20- 0	Uttarābhādrā	11- 7- 0
Chitrā	6- 3- 0	Revatī	0- 0- 0

Comm. In the enumeration of these longitudes, Bhāskara makes three statements which we have to note (1) That these longitudes are rectified for Āyana-Dṛk-Karma ie. that they are polar longitudes, (2) That the longitudes of Kritticā and Rohinī are less by 32′. (The longitudes shown in the table above are those incorporating this specified correction), (3) That the longitudes of Vis'ākhā, Anūrādhā and Jyeṣthā are to be increased by 5′. (This correction is also incorporated in the table given above).

It is to be noted that these longitudes are the same given by Brahmagupta originally and copied by S'rīpati as well as the corrections indicated above. But herein a mistake was committed as clarified by Bhāskara later in the end of the chapter namely that the polar longitudes are subject to what is called Āyanavikāra though the celestial longitudes are not. We say that the celestial longitudes are not subject to such an Āyanavikāra ie. that change due to the phenomenon known as the precession of equinoxes, because the Hindu system is Nirayana ie. reckons the longitudes from the first point of As'winī which is its zero point instead of reckoning from *r*. If longitudes are measured from *r*, as *r* is preceeding, the longitudes of stars will be steadily increasing all at the same annual rate of precession namely about 50–25″ per year. These increasing longitudes of the modern system go by the name Sāyana longitudes. It might be thought that since the polar longitudes also are measured from As'winī and along the ecliptic like celestial longitudes, they also don't vary like the Nirayana celestial longitudes; but it is not so, because the effect of precession on the right ascension and declination of a star have both their effect upon the polar longitude as well as polar latitude. We cannot say that Bhāskara did not know this but we may say that the effect on the polar longitude and latitudes were construed by him as neglible and would be appreciable only in the long run.

Verses 4, 5, 6. Sphutas'aras or rectified latitudes of the stars.

As'winī	10°– 0 north	Ārdrā	11 – 0 south
Bharaṇī	12°– 0 ,,	Punarvasū	6 – 0 north
Krittica	4 –30 ,,	Puṣyamī	0 – 0 ,,
Rohinī	4 –30 south	Asreṣā	7 – 0 south
Mṛgas'irṣa	10 – 0 ,,	Makhā	0 – 0 north

Purvāphalgunī	12 – 0 north	Purvāṣādhā	5 –20 south
Uttarāphalgunī	13 0 „	Uttarāṣādhā	5 – 0 „
Hasta	11 – 0 south	Sravaṇam	30 – 0 north
Chitrā		Abhijit	62 – 0 „
Swātī	37°– 0 north	Dhaniṣṭha	36 – 0 „
Vis'ākhā	1 –20′ south	S'atabhiṣak	0 –20 south
Anūrādhā	1 –45 „	Purvābhādrā	24 – 0 north
Jyeṣṭhā	3 –30 „	Uttarābhādrā	26 – 0 „
Mulā	8 –30 „	Revatī	0 – 0 „

(*a*) These are also what were given by Brahmagupta and copied by S'rīpati, (*b*) Bhāskara mentions in the

" ie. In as much as the stars are fixed we have given their latitudes and longitudes rectified. But here, there is a point to be noted as Bhāskara has put us in a doubt namely that the rectified latitudes which he speaks of elsewhere under verse (3) Grahacchāyādhikāra, Ganitādhyaya are not exactly the polar latitudes spoken of here. (Ref. fig. 120). Let γMR be the ecliptic and γN be the celestial equator. Let S be a star, *k* be the pole of the ecliptic, and *p* be the celestial pole. Then γR is the celestial longitude, SR the celestial latitude which are known as the Dhruvaka and S'ara (Asphuta-s'ara). γM is the polar longitude and SM the polar latitude. MR is the arc connoting the Āyana-Dṛk-Karma correction, so that celestial longitude ± Āyana-Dṛk-Karma correction = polar longitude (plus or minus according as the longitude lies in the 2nd and 4th quadrants or 1st and 3rd quadrants). If the longitude just equals 0°, 90°, 180° or 270°, the correction of

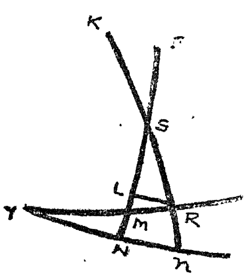


Fig. 120

Karma vanishes. It must be noted here that when the longitude is 0° or 180° Āyanavalana is maximum but Āyana-Dṛk-Karma vanishes. There is no ambignity here in this polar longitude because Bhāskara is unequivocal in defining this. But under verse (3) Grahacchāyādhikāra, Bhāskara means by Sphutas'ara SL and not SM but by Sphutas'ara here he means SM. It is ridiculous to suppose that Bhāskara did not know that $\beta\sqrt{R^2-\text{Āyanavalana}}$ is less than β. This $\beta\sqrt{R^2-a^2}/R$ (a=Āyanavalanajyā) he defined as Sphutas'ara there under verse (3) cited. SR is defined by him as Asphutas'ara or simply S'ara. Now here in the commentary he makes us believe that Sphutas'ara is SM which is the polar latitude. This confusion created by Bhāskara leads Ramaswarup the editor of Brahmagupta Siddhānta as well as one Mukhopadhyaya the author of the thesis, 'Hindu Nakshatras' to suppose that Bhāskara was wrong in supposing that $\frac{\beta}{R}\sqrt{R^2-a^2}>\beta$. Bhāskara could not evidently commit such a silly mistake but we must infer that in that context he called SL as the Sphutas'ara which being added to Rn the Krānti or what is the same LN gives SN the Sphutakrānti or the modern declination. Rn is called by him as Asphutakrānti. In this context he calls SM as the Sphutas'ara since it is Dhruvābhimukha ie. directed towards the pole. Of course, Bhāskara should not have called both SL and SM as Sphutas'aras but since he was deliberately defining the Sphutas'ara as SL previously and now as SM, and since he could not commit such a glaring mistake as to construe $\frac{\beta}{R}\sqrt{R^2-a^2}$ as greater than β, we should not rush to pronounce that Bhāskara was wrong. Only we could say that he is inconsistent to that extent.

7. The polar longitudes of Agastya (Canopus) is 87° and his polar latitude is 77 south. The polar

longitude of Lubdhaka (Sirius) 86° and its polar latitude is 40° south.

Comm. Clear. The star Agastya is considered to be important in Indian literature because the heliacal rising and setting of Agastya are directed to be noted and in fact are being noted in every panchanga even today from times immemorial. Even Kālidāsa alludes to this heliacal rising of Agastya as inaugurating the S'arat-kāla or autumn which was reiterated by Varala Mihira in his Bṛhat-Samhitā under the verse "भगवति जलधरपक्ष्मक्षपाक्रराकेंक्षणे कमलनाभे, उन्मीलयति तुरङ्गमकरिरथनीराजनं कुर्यात्" ie. when Lord Vishnu whose eyes are supposed to be the Sun and the Moon and his eyelids the clouds, opens his eyes ie. on Kārtica Ekādasi the 11th day of the bright half of the lunar month of Kārtica then Kings are directed to perform Nīrājanavidhi for his horses, elephants and chariots (to start on an expedition for war). It is to be noted that this was so long ago in times of yore that Agastya used to rise at the beginning of autum. Now the star is rising about 22nd August long in advance even in the rainy season. This is on account of the effect of precession of equinoxes.

Verse 8. The Iṣtanādis for Agastya are said to be two, for Lubdhaka $2\frac{1}{6}$, for other stars which are next in size, $2\frac{1}{3}$ and for still smaller ones the Iṣtanādis are to be taken still more.

Comm. Iṣtanādis ie. the time in nādis (where a is equal to 24′ of time) giving the time in between the rising moments of the star and the Sun, when the star rises heliacally. In other words, let the star Agastya rise at a particular time t. If the Sun rises at $t+48'$, then it is the time for Agastya to rise heliacally. This again means that if the Sun sets at time T and the star at T+48′ in its diurnal motion, it is time for the star to set heliacally in the west. Similarly for the other stars. It will be noted here that stars set heliacally in the west and rise heliacally in the east. This phenomenon has been long in the notice of even the most illiterate people of India from times immemorial, as they were used to get up from beds round about the time when a brilliant star rose heliacally and stood in the eastern horizon, shining for a good length of time before Sun-rise. Each star of first magnitude thus played the part of a morning star for some time, though perhaps the illiterate folk mistake them to be the same star. Especially the brightest stars of the zodiac thus play the part of morning and evening stars. The case with respect to Agastya and Lubdhaka is different in that even though they are far away from the zodiac, yet they were noticed to be morning and evening stars by virtue of their being of the first magnitude at a spot of the sky in the vicinity of which no other such brilliant stars are there. This is the reason why panchanga—Computers have been in the habit of recording in the panchanga even to-date the heliacal rising and setting of Agastya, also because the heliacal rising of this star synchronized with the setting in of S'arat-kāla or autumn. Since in India the lunar Kārtica Ekādasi, ie. the 11th day of the bright half of the lunar month roughly synchronized with 15th Nov., when the Sun rose far in the south of the horizon, this Agastya, though it be in the far south, happens to play the part of a morning star and about the day when it rose heliacally, there were no more rains and waters of the rivers stood crystal-clear as described by many a Sanskrit poet like Kālidāsa (vide the famous verse of Kālidāsa प्रससादोदयादम्भः कुम्भयोने र्महौजसः 4th canto Raghu-vams'a).

The star Lubdhaka or Sirius, the dog-star as it is called in English parlance also was conspicuous as a morning and evening star, whose heliacal rising was noticed and recorded by English poets like Shakespeare and Milton.

Since one nādi corresponds to 6°, two nādis correspond to 12°, degrees, which are spoken as Kālāms'as for the heliacal rising of Agastya. They are so termed, because they indicate the Kāla or the time in between the rising of the star and the Sun which signifies the moment of its heliacal rising. Indirectly therefore these Kālāmsas indicate which star is of which magnitude. A star which has 12° as Kālāmsas is therefore of first magnitude; and as the Kālāmsas increase, the magnitude also increases. It will be rembered that the higher the magnitude of a star, the fainter it will be and not the brighter as is likely to be misconstrued by lay people.

Verse 9. To compute the moment of conjunction of a planet and a star.

The Āyana-Dṛk-Karma is to be done as mentioned before (with respect to the planet) and the Sphutaśara is to be computed to know the time of (polar latitudinal) conjunction.

Comm. We are directed to use the polar longitude and polar latitude with respect to the planet because this kind of conjunction will be more conspicuous than a celestial latitudinal conjunction because the Ecliptic is far more inclined than the Equator with respect to the horizon.

Verses 10, 11.

The difference in the longitudes of the planet and the star, divided by the daily motion of the planet, gives the number of days approximately after or before the moment of conjunction.

If the planet be retrograde, the conjunction past or future will be in the reverse ie. future or past.

Comm. Let x and y be the polar longitudes of the planet and the star and let $x<y$. Since y is constant as the star has no motion, x has to increase to the extent of

y to be in a polar latitudinal conjunction. Hence $(y-x)$ is to be covered by the planet as per its daily motion say m. So the number of days that is to elapse for conjunction is $\frac{y-x}{m}$. If $x>y$, then the conjunction was past by $x-y/m$. If, the planet be retrograde and $x<y$, the conjunction was past by $\frac{y-x}{m}$ days and if $x>y$, the planet being retrograde, the conjunction is to take place in $\frac{x-y}{m}$.

Note. Though Bhāskara does not mention here Asakṛt-Karma ie. method of successive approximation, it is implied because the motion of the planet differs from moment to moment as well as its Sphutas'ara. Hence having obtained the approximate moment of conjunction, compute again the true motion of the planet at that instant, as well as its Sphutas'ara and Āyana-Dṛk-Karma. The latter ie. Āyana-Dṛk-Karma is to find the polar longitude from the celestial and the Sphutas'ara is the polar latitude ie. SM of fig. 120. The Sphutas'ara is required for the purpose of finding the distance in between the planet and the star on a polar latitudinal circle and not to compute the moment of conjunction.

Verses 12, 13, 14. To compute the moments of heliacal rising and heliacal setting of a star.

Compute the Udayalagna and the Astalagna of Agastya and Lubdhaka doing Ākṣa-Dṛk-Karma alone. Assuming the Udayalagna to be the Sun, compute the lagna for the Iṣta-kāla nādis (ie. after a lapse of Iṣtanādis after the moment) given before (namely 2 nādis for Agastya and $2\frac{1}{6}$ for Lubdhaka) which will be the longitude of the Sun, when the star (Agastya or Lubdhaka or whatever it be) rises heliacally. Thus the Udayārka is a point of the ecliptic which rises when the star rises heliacally.

Having computed the Asta-lagna of the star, taking it to be the Sun, compute the lagna in the reverse direction ie. the lagna which preceeds it by the Iṣṭa-kāla given. If this lagna be decreased by 180°, it will give the longitude of the setting Sun at the time of the heliacal setting of the star. Or again find the longitude of the point of the ecliptic which is ahead of the Astalagna which takes (60—Iṣṭanādis) to rise after the Astalagna; this longitude decreased by 180° gives the longitude of the setting Sun at the time of the heliacal setting of the star. The heliacal rising or setting takes place when the longitude of the Sun equals the longitude of the point of the ecliptic which is technically called the Udayārka or the Astārka respectively. The difference between the longitude of the Sun and that of the Udayārka or the Astārka divided by the daily motion of the Sun gives approximately the number of days that have elapsed or are to elapse for the heliacal rising or setting as the case may be,

Comm. Here we are to carefully differentiate between the technical words (1) Udayalagna of the star, (2) Asta-lagna of the star, (3) Udayārka and (4) Astārka of the star. The word Udayalagna means that point of the ecliptic which rises simultaneously with the star. The word Astalagna means that point of the ecliptic *which is rising* when the star is setting. The word Udayārka means that point of the ecliptic which rises when the star rises heliacally. The word Astārka means that point of the ecliptic *which is setting* while the star sets heliacally. Thus the four points are only points of the ecliptic.

Let A be the Udayalagna of a star on the circle CAB which is the ecliptic. We know that the position of the star should be above the eastern horizon, to rise heliacally, by such a distance that the time in between the rising of the Udayalagna and that point of the ecliptic which will be rising when the star rises heliacally must be the Iṣṭa-kāla nādis. Let B be a point ahead of A on the ecliptic

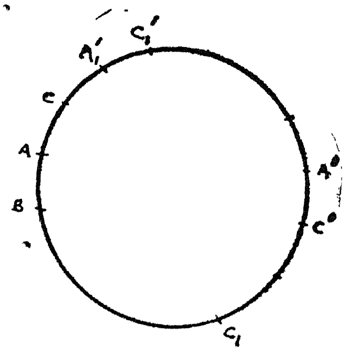


Fig. 121

such that the time in between the rising of B and the rising of A is equal to the Ista-nādis. By the time B rises, A will have gone up a little above the eastern horizon, as well as the star that has risen along with A and now the position of the star is such that the time in between its rising and the rising of B is equal to the Iṣta-nādis. Hence B, which is a point of the ecliptic is termed Udayārka signifying thereby that when the Sun coincides with B, the star rises heliacally. Thus the Udayārka B is ahead of the Udayalagna A and the time in between their risings is equal to Iṣta-nādis.

Let now A′ be the Astalagna ie. the point which rises when the star sets. This point will not be, generally, diametrically opposite to A because, the time between the rising and setting of a star which is given by double the rising hour-angle will be far more than half a sidereal day when the star has a large northern latitude and in the case of one having a large southern latitude, the time will be far less than half a sidereal day.

From A′ find C′ which is behind A′ such that the time in between the rising of A′ and C′ is equal to Iṣta-nādis. Then the point C diametrically opposite to C′ namely C is called Astārka ie. when the Sun is at C the star sets heliacally in the west. It is so because, when the Sun is on the western horizon setting at C, C′ will be the lagna and when the star is setting A′ is the lagna and as these two lagnas differ by Iṣta-nādis, the time between the Sun's setting and the star's setting is also equal to Iṣta-nādis.

In other words the star is within the Iṣṭa-nādi distance from the Sun and as C is behind A by Iṣṭa-nādis, the setting Sun at C will be behind (ie. has a lesser longitude) the setting star by Iṣṭa-nādis. Hence the star sets then heliacally. Thus we see that the Udayārka and Astārka given by the points B and C are respectively in advance (ie. has a greater longitude) and behind (ie. has a lesser longitude) of the Udayalagna and Astalagna removed by an Iṣṭa-nādi distance. It will be noted that the arcs AC and AB are not equal though the equatorial arcs corresponding to them are equal.

Instead of finding C′ from A′ by the Vilomalagna method and taking its diametrically opposite point, Bhāskara gives an alternative namely to find C from A′ by the Kramalagna method the time being (60–Iṣṭa-nādikas). This is evident.

Bhāskara prescribes only Ākṣa-Dṛk-Karma to be performed here in finding the Udayalagna and Astalagna of the star because the star's polar longitude is already one in which the Āyana-Dṛk-Karma is contained.

Thus from fig. 121, when the Sun-rises at A, the star rises, when the Suns rises at B the star rises heliacally and when the Sun sets at C, the star sets heliacally.

Verse 15. If in the case of a star, the Astārka happens to have a longitude greater than the Udayārka on account of a very big northern latitude, that star does not set heliacally (and so the question of heliacal rising does not arise).

Comm. In Fig. 121, the Astārka C happens to have a longitude less than that of B (because CAB is the direction of increasing longitude ie. positive direction). At times it so happens that C lies towards the positive direction of B ie. it has a longitude greater than that of B.

This happens when the star has a long northern latitude and therefore the Ākṣa-Dṛk-Karma correction will be sufficiently large and the star has a small north polar distance. This may be substantiated as follows.

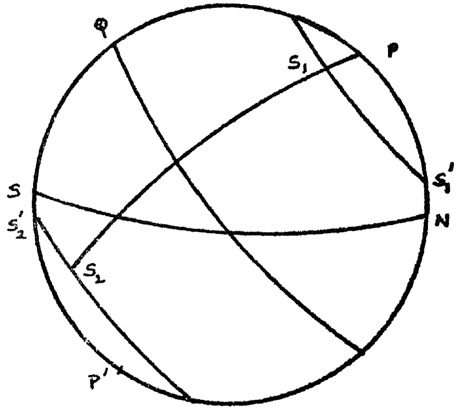


Fig. 122

We have the formula $\cos h = -\tan \phi \tan \delta$ which gives the rising hour-angle of the star. When δ is nearly equal to $90-\phi$, then $\cos h$ will be nearly equal to '—1' which means that the rising hour angle is nearly equal to 180°, ie. the duration of the star's stay below the horizon will be very small. This means A′ approaches A very nearly (fig. 121), for example when it is in the position A_1'. Then let C_1' be the point which is behind A′ by Iṣṭa-nādis so that C_1' the diametrically opposite point C_1 is far ahead of B in stead of preceeding it. This means that the star which should first set heliacally and then after a few days rise heliacally, is now in such a position that it is to rise heliacally even before setting heliacally. This is an untenable position. That this is not tenable may be seen otherwise. The diametrically opposite point of Astalagna ie. the point C of the ecliptic which should set along with the planet should have a longitude less than that of the

Astārka C_1'; but C_1' will have now a longitude less than C which is incongruous.

In such a situation, Bhāskara says, there is no question of the star setting heliacally at all. This is indeed, an ingenious mathematical presentation of a physical phenomenon, which reflects credit to Bhāskara's genius.

Verse 16. Circumpolar stars or Sadōdita stars.

If a star has a northern declination greater than $90-\phi$ (ie. $\phi>\overline{90-\delta}$) it will be always above the horizon; also if the southern declination is greater than $\overline{90-\phi}$ such a star will never be seen in a northern latitude, be it Lubdhaka or Agastya or even a planet for the matter of that.

Comm. (Vide fig. 122) Let S_1 be a star such that $PS_1'<PN$ ie. $90-\delta<\phi$ ie. $\delta>90-\phi$. It is clear from the figure that its diurnal path is entirely above the horizon. It is called a Sadōdita star or circumpolar.

Take the case of S_2. Its southern declination ie. QS_2' is greater than the lamba $(90-\phi)$ ie. QS. It is evident from the figure that its diurnal path is entirely below the horizon.

Bhāskara gives two examples here namely (1) where the latitude is greater than 37°, there Agastya will not be visible (having a great north-polar distance), (2) where the latitude is greater than 52°, thēre Abhijit is always above the horizon (having a small north-polar distance).

Bhāskara adds, 'even a planet'. This will be true, for example, in a high latitude say 89°. Suppose the southern declination of a planet is greater than 1°. On that day and for some more days also, the planet's diurnal paths will be below the horizon as will be clear from a figure.

Verses 17 to 21. Ancient astronomers happened to give a list of polar longitudes and polar latitudes at a time when there were no Ayanāmsas ie. when the zero-point of the ecliptic as taken by the Hindu Astronomers namely Aswini coincided with the modern zero-point namely *r*.

In fact these polar longitudes and latitudes do change if there be Ayanāmsas. Here in this case from the polar latitudes the celestial latitudes are to be computed in a reverse process; with the half of these celestial latitudes, the Āyana-Dṛk-Karma is to be effected in the reverse process to obtain the celestial longitudes. After having obtained the correct celestial longitudes and celestial latitudes, now, bringing the Ayanāmsas into the picture, compute the correct Dṛk-Karma and also rectify the celestial longitudes, to obtain the correct polar longitudes and polar latitudes to compute the moment of polar latitudinal conjunction. This case may be taken when the Ayanāmsas are large, otherwise, a small difference there will be, (which does not matter).

Comm. Bhāskara gives the process in the course of the commentary. We have the formula

$$\text{Sphuta Vikṣepa} = \frac{\text{Asphuta Vikṣepa} \times \text{Yaṣṭi}}{\text{Radius}}$$

Here we know Sphuta Vikṣepa. At once, we could not say $\text{Asphuta Vikṣepa} = \frac{\text{Sphuta Vikṣepa} \times \text{Radius}}{\text{Yaṣṭi}}$ computing Yaṣṭi from the modern Ayanāmsas. Yaṣṭi is a function of declination too, because the Āyanavalana is a function of declination. We know, the declinations change in the wake of precession of the equinoxes. So, there is no point in taking the value of the present Yaṣṭi. We should compute it for the Āyanas'ūnyakāla or for the time when As'wini coincided with *r*. Then the formula could be applied. Then with this celestial latitude obtained, Āyana-Dṛk-Karma is to be effected to obtain the present polar longitude, using the modern Ayanāmsas. Since in this process, we have no definite knowledge of the then celestial longitude ie., of the Āyanaśūnyakāla, the method of successive approximation is appealed to.

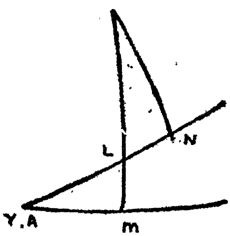


Fig. 123

But this method of Bhāskara may be modified to an easier process as follows. Let rL, LS be the polar longitude and polar latitude of a star as given by Acharyas in whose time the Hindu first point of the zodiac coincided with r. It is required to find rN and NS the celestial longitude and latitude which hold good even today becaus rN and NS are the same as AN, AS, A being the first point of Aświni and a celestial longitude measured from A along the ecliptic is not subject to change on account of precession of the equinoxes as well as the celestial latitude. From the spherical triangle rLM, $\cot r\mathrm{LM} = \cos r\mathrm{L} \tan \omega$ (1) and from the triangle SNL, $\tan \mathrm{LN} = \cos \widehat{\mathrm{SLN}} \times \tan \mathrm{SL} = \cos r\mathrm{LM} \tan \mathrm{SL}$ (4) and $\sin \mathrm{SN} = \sin \mathrm{SL} \times \sin \mathrm{SLN} = \sin \mathrm{SL} \times \sin r\mathrm{LM}$ (5). From (1) the angle $\widehat{r\mathrm{LM}}$ is obtained; substituting this value in (2) and (3) LN and SN are obtained. Adding LN to rL we have rN.

Thus the celestial longitude and latitude are found far more easily and more accurately than from the laborious method indicated which gives only approximate results.

Here ends the Bhagrahayutyadhikāra.

PĀTĀDHYĀYA

Verse 1. Even scholars get confused while computing the occurence of a Pāta, unable to know whether it has already occurred or is going to occur. So, I seek to clarify the method of computing the moment of occurrence of a pāta.

Comm. (1) There are what are called pātās, two in number, called Vyatipāta and Vaidhṛti. The first is defined as occurring at that moment when the Sun and Moon have equal declinations, being situated in opposite Ayanas but the same goḷa. The words Uttaragoḷa and Dakṣinagoḷa are used by Hindu Astronomers as the northern and southern halves of the celestial sphere on either side of the celestial equator, respectively; where as the words Uttara–Ayana and Dakṣiṇa–Ayana are used to connote the times when the Sun or Moon have tropical longitudes (measured from *r* instead of Aswini, the zero point of the Hindu Zodiac) one lying between Capricorn and Cancer, and the other lying between Cancer and Capricorn. Thus Vyatipāta occurs when the Sun and Moon each lies in one of the first or second quadrants of the ecliptic measured from *r* or each lies in one of the third or fourth quadrants of the ecliptic; (both should not lie in the same quadrant) and have equal declinations.

The second Pāta Vaidhṛti occurs when the Sun and Moon have equal declinations, they being situated in the same Ayana but opposite goḷas. These two moments are considered to have malefic effects on humans and the world of life. Though the moments are to be computed by a knowledge of spherical astronomy, they have only an astrological significance. A chapter, usually the last, has been devoted to this subject in every book of Hindu Astronomy. The method of computation was felt difficult by the ancient Hindu astronomers, before the time of Bhāskara, for the reason that the Moon does not exactly move in the ecliptic but in his own orbit whose inclination to the ecliptic was taken to be 4½°. The points of intersection of the ecliptic with the lunar orbit, or what are called Rāhu and Ketu, the nodes of the lunar orbit, have themselves a motion backwards on the ecliptic in 18.59575 solar years as per Bhāskara. This makes the lunar orbit oscillate about the mean position of the ecliptic, so that sometimes the lunar orbit lies between the celestial equator and some-times not between them. This phenomenon makes the computation more difficult, to obtain the declination of the Moon. Bhāskara says that even great Ācāryas like Lalla and Brahmagupta went wrong or got confused in computing the moments of the occurence of a Pāta. Bearing upon Bhāskara's statement that such Ācāryas also got confused, some second-rate astronomer exclaimed in the following interesting manner "त्रिस्कन्धविद्याकुशलैकमल्लो

भूत्र, पि किञ्चि

ie. "when even an unrivalled scholar like Lalla, who was well-versed in the three branches of Jyotiṣa betrayed his ignorance in this context of Pātādhikāra, though I am a bit of a mathematician, I could have no pretensions to any knowledge in this difficult chapter".

In Fig. 124, Vyatipāta occurs when SL=MN, where S and M are the Sun and the Moon situated respectively in Uttarāyaṇa and Dakṣiṇāyana but in the same uttaragoḷa. The positions of S and M could be interchanged ie. S may be in the second quadrant of the ecliptic and M in the first quadrant and if their declinations be equal, then also Vyatipāta occurs. Of course in this figure 124, we have taken the Moon also situated on the ecliptic. In

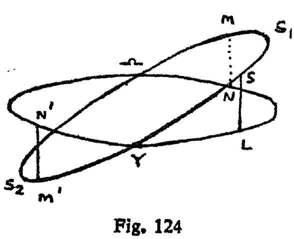


Fig. 124

actual computation, we should not take the Moon as such. The pāta named Vaidhṛti occurs when in the same Fig. 124, $SL=M^1N^1$. Herein both the Sun and Moon are in the same Ayana namely Uttarāyaṇa but in opposite goḷas. If we assume the Moon to be moving on the ecliptic, Vyatipāta occurs when the sum of the tropical longitudes of the Sun and Moon equals 180° and Vaidhṛti occurs when the sum of those longitudes is equal to 360.

Verse 2. Definitions of Goḷa-Sandhi and Ayana-Sandhi with respect to the Sun.

When the tropical longitude of the Sun ie. his longitude measured from ♈ along the ecliptic is equal to 0° or 180°, then he is said to be at his goḷa Sandhi. In other words, when the Sun who moves along the ecliptic comes to the celestial equator, he will be at his Goḷa sandhi. Similarly when his longitude is 90° or 270°, he is said to be at the his Ayana-Sandhi.

Comm. This means that when the Sun is about to pass from the Dakṣiṇa goḷa to the uttaragoḷa or from the uttaragoḷa to the Dakṣiṇagoḷa he is said to be at a Goḷa Sandhi. Similarly when he is about to go south or when he is about to go north, he is said to be at Ayana Sandhi. The word 'Sandhi' means junction. Thus the points ♈ and ♎ (Libra) are said to be goḷa Sandhis whereas the points denoting Cancer and Capricorn are Ayana-Sandhis. Since in Bhāskara's time the Ayanamsas were 11°, ie. the point ♈ was behind the zero point of the Hindu Zodiac by 11°, therefore Bhāskara gives the Goḷa Sandhis as the points having longitudes 349° and 169° respectively. Similarly the Ayana Sandhis were the points having longitudes 79° and 259° respectively.

Bhāskara gives the method of locating these goḷa or Ayana Sandhis as follows. Erect a gnomon vertically with the help of a plumb-line. Draw a circle on the horizontal plane having the foot of the gnomon as the centre and any arbitrary radius. Draw the East-West and North - South diameters of the circle. The longitudes of the Sun when the shadow of the gnomon lies along the East - West diameter give the goḷa Sandhis. Note the direction of the shadow daily after the day when his shadow is along the East - West line. If the Sun rises thereafter a little towards North of the East point, then he is said to be in the uttara goḷa; his shadow will be a little south of the East - West line. The point at which the Sun thus begins to rise north of the east point, is the Meṣa goḷa Sandhi or the vernal equinox. Gradually the Sun goes on rising at points which are farther and farther away from the East point. When he has reached the extreme north point, ie when the gnomon's shadow is extreme south, he is at the Cancer. Similarly when he rises at the extreme South point on the horizon, he is at Capricorn. Bhāskara actually noted these four points and the longitudes cited above were given by him as the Goḷa Sandhis and Ayana Sandhis. Indirectly, he could know that the Ayanāmśas at the time of his writing the book, were 11°.

Verses 3 to 6. Speciality with respect to the Moon.

Let the Hsine and Hcosine of the tropical longitude of the pāta (Rāhu the ascending node of the lunar orbit) as per the smaller table of Hsines when the radius is taken to be 120′, be respectively multiplied by 123 and 7 and divided respectively by 4 and 12. The results are known as the Bāhuphala and Koṭiphala respectively. According as the tropical longitude of Rāhu ie λ be such that as $270° < \lambda < 90°$ or $90° < \lambda < 270°$, the Bāhuphala is to be divided by 362 $\pm$ Kotiphala. Subtract the result from the Goḷa Sandhis and Ayana Sandhis of the Sun, to get those of the Moon.

Comm. from the Hindu Astronomical point of view this is an intricate procedure which made that particular half-learned astronomer declare "पाताधिकारे मम नाऽधिकारः" ie. "I have no pretensions to have understood the chapter known as Pātādhikārā", We perceive herein Bhāskara's mathematical understanding of the problem. His procedure is approximate because he uses (1) the smaller crude table of Hsines taking the radius to be 120′ instead of 3438. (2) also because he gives the Sun's declinations for longitudes 15°, 30° etc. as well as Moon's celestial latitudes for his longitudes of 15°, 30° etc. relative to the node in his orbit. Of course, his mathematical procedure was correct.

He exemplifies his mathematical procedure by solving a problem given in the pras'na–Adhyāya of his book Goḷādhyāya. The problem set by him is as follows.

नांशोऽस्त्रशतं शशी

ie. If the tropical longitude of the Moon is 100°, that of the Sun 80°, and the longitude of the Moon measured in his orbit from the Node Rāhu is 200°, compute the moment of the occurence of the pāta, if you know what was said by Lalla in his work S'iṣya dhīvṛddhida. We shall first understand Bhāskaṛa's mind and subsequently we shall a modern procedure.

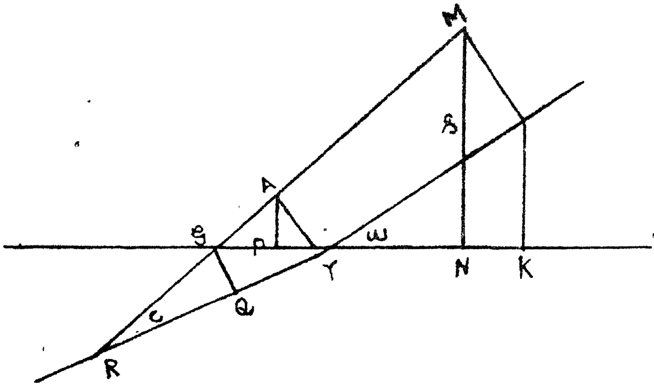


Fig. 125

Refer to fig. 125. Gr NK is the celestial equator. RrL is the ecliptic and RGAM the Moon's orbit where R is the Rāhu or ascending node of the lunar orbit. Let the obliquity of the ecliptic be ω (omega) and the inclination of the lunar orbit to the ecliptic be i. ω was taken to be 24° and i $4\frac{1}{2}$° by Bhāskara. Let the lunar orbit RGAM cut the celestial equator in G which is called the Moon's Goḷa Sandhi. r is the Sun's Goḷa Sandhi. Bhāskara first wants us to locate G ie. to find rQ which gives its longitude. Let A be the position of the Moon when his celestial longitude is zero ie rA is perpendicular to the ecliptic. Let M be the position of the Moon when his celestial longitude is 15° (Here the figure is not drawn to scale but a little exaggerated for the sake of clarity). Let ML be the celestial latitude of the Moon in its position. M. If LK be drawn perpendicular to the equator, LK is called the Asphuta-krānti of the Moon. ML is called the Asphuṭa-Vikṣepa of the Moon. MN drawn perpendicular to the celestial equator ie. the declination of the Moon is called Sphuṭakrānti of the Moon. Draw perpendicular LS on MN. Then MS is called the Sphuṭa Vikṣepa of the Moon. Since MN = Ms + LK Sphuṭakrānti = sphuṭa Vikṣepa + Asphuṭakrānti. Let Ap be the declination of the Moon when his longitude is zero.

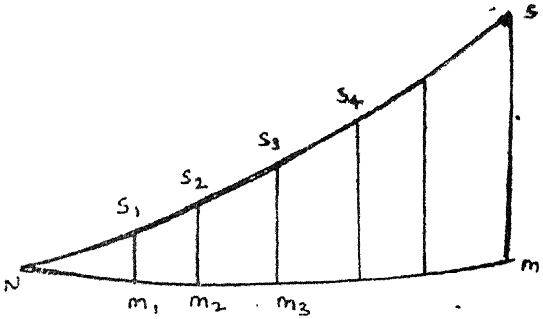


Fig. 126

Now refer to fig. 126. Let NS be the ecliptic and NM the celestial equator. The quadrant NS equal to 90° is divided into six equal parts at S_1, S_2 etc. The successive declinations of S_1, S_2 etc. are given by Brahmagupta as 362, 703, 1002, 1238, 1388, 1440 where the radius is given to be 3438′. In other words 1440 = H Sin ω = H Sin 24°. These can be easily verified from the formula

$$\text{H Sin } \delta = \lambda \times \text{H}$$

putting λ = 15°, 30° etc. and R = 3438′.

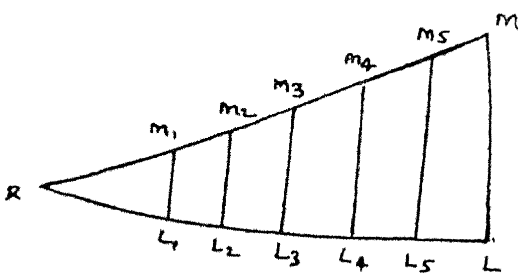


Fig. 127

Now refer to fig. 127. R M is the lunar orbit, R being Rāhu. RL is the ecliptic. Using the formula.

$$H \sin \quad \frac{H \operatorname{Sin} \lambda \times H \operatorname{Sin} i}{R} \quad \frac{H \operatorname{Sin} \lambda \times i}{120}$$

since β and i are small and R is taken to be 120′, we have the successive values of M_1L_1, M_2L_2 etc. of the celestial latitudes of the Moon for arcs RM_1, RM_2 etc. successively equal to 15°, 30° etc. given by 70, 135, 191, 234, 261, 270. In other words the maximum celestial latitude is $270' = 4\frac{1}{2}°$.

Since when $\lambda = 15°$, taking $R = 120$ $\beta = 70'$, the first S'arakhanda ie the celestial latitude for $\lambda = 15°$, is 70′ $\frac{H \cos \lambda \times 70}{120}$ Bhāskara Calls Koṭiphala.

It will be noted here that the first S'arakhanda is taken to mean the increase in the celestial latitude from zero to 70′ when the longitude increases from 0° to 15°. The argument adduced by Bhāskara in such a context is as follows: "If for a H Cosine λ equal to R equal to 120′ (when $\lambda = 0$) we have the first S'arakhanda namely 70′, what shall we have for an arbitrary H Cosine λ". The result is $\frac{H \cos \lambda \times 70}{120} = \frac{7}{12} H \cos \lambda$. Since the word Cosine means Koṭijyā, Bhāskara calls this Kotiphala.

Now in fig. 125, let $\gamma L = 15°$, then LK = 362′ as given by Brahmagupta. Bhāskara takes this 362 as Δ (δ) where δ is the declination of the foot of the celestial latitude of the Moon. Then if β be the celestial latitude of the Moon, Bhāskara construes that $\Delta \beta$ is given by the formula $\frac{7}{12}$ H Cos λ, which he calls Kotiphala. Taking $\Delta\delta \pm \Delta\beta$ as the joint variation of δ and β which is roughly equated with the variation in the modern declination MN of the Moon, Bhaskara's argument is "If for a longitude γ M = 15° of the Moon corresponds $\Delta\delta \pm \Delta\beta$ what should be the longitude corresponding to the declination Ap of the Moon when his longitude is equal to zero?" The result is

$$\frac{Ap \times 15}{\Delta\delta \pm \Delta\beta} = \text{II}$$

where rQ is looked upon as the longitude of the Moon at G called the Goḷa Sandhi of the Moon.

But AP = the Sphuṭa Vikṣepa of the Moon when his longitude is equal to zero where rA is the Asphuṭa Vikṣepa at that point. Using his method of calculating AP from Ar, $Ap = \frac{Ar \times H\cos\delta}{R}$ where

$$H\sin\delta = \frac{H\sin\omega \times H\sin(90 + \lambda)}{R}.$$

AP pertains to the longitude λ equal to zero, so that $H\sin\delta = \frac{H\sin 90° \times H\sin\omega}{R} = H\sin\omega.$

$\therefore$ $H\cos\delta = H\cos\omega = H\cos 24° = 110$ when $R = 120'$

Hence $Ap = \frac{Ar \times 110}{120} = \frac{11}{12} Ar.$

But Ar is the celestial latitude of the Moon to be calculated from RA taken to be λ

$\therefore$ $rA = \frac{H\sin\lambda \times 270}{120} = \frac{9}{4} H\sin\lambda$ (where $270' = 4\frac{1}{2}° = i$)

taking $R = 120$ $\therefore$ $Ap = \frac{9}{4} H\sin\lambda \times \frac{11}{12}$. Hence

rQ from I $= \frac{9}{4} H\sin\lambda \times \frac{11}{12} \times \frac{15}{\Delta\delta + \Delta}$

But $\frac{H\sin\lambda}{4} \times \frac{135 \times 11}{12} = \frac{123}{4} H\sin\lambda$. This has to be divided by $\Delta\delta \pm \Delta\beta$. In the problem given. $\lambda = 100°$ and $\Delta\delta$ is taken as 362. To obtain $\Delta\beta$ Bhāskara adduces the argument "If at $\lambda = 0$, the first S'arakhanda of 70 corresponds to $H\cos\lambda = 120'$ what amount of S'arakhanda corresponds to an arbitray $H\cos\lambda$?" The result is $\frac{H\cos\lambda \times 70}{120} = \frac{7}{12} H\cos\lambda$. This he calls Kotiphala because it is based upon $H\cos\lambda$ which is the Kotijyā.

Thus in the given problem where $\lambda = 100°$,

H cos 100° $= \frac{7}{12} \times 21$ (since H cos 100 = − H sin 10 = − 120 × ·1735 = − 21 approximately where R = 120′) = − 12′-15″ $\therefore\ \Delta\delta \pm \Delta\beta = 362 - 12'\text{-}15 = 349\text{-}45$.

Now $\frac{123}{4}$ H sin λ to be divided above $= \dfrac{123 \times 118}{4}$

$= 3628'\text{-}30''$,

so that $\frac{123}{4}$ H sin $\lambda \times \dfrac{1}{\Delta\delta \pm \Delta\beta} = \dfrac{3628'\text{-}30''}{349\text{-}45}$

$= 10°\text{-}22'\text{-}28'' =$

$= r$Q. Now r's longitude from the zero point of the Hindu zodiac is 11 Rāśis 19° so that Q's longitude = 11 R-19° minus 10°-22′-28″ = 11 Rāśis 8°-37′-32″. This gives us the longitude of G which is called the Moon's Goḷa Sandhi. Adding 3, 6, 9 Rāśis, we get successively the first Āyana Sandhi, the other Goḷa Sandhi and the second Āyana Sandhi of the Moon.

(1) Bhāskara overlooked a crudeness in his procedure namely that $\Delta\beta$ is perpendicular to the ecliptic whereas $\Delta\delta$ is perpendicular to the celestial Equator, but, since $\Delta\beta$ is small, he overlooked the nicety. Strictly speaking $\Delta\beta$ should have been corrected for Āyanavalana.

(2) There is also crudeness in computing or arcs of 15°, whereas they should have been done for increase of every degree in the longitude. He could have done that, because at the end of the Goḷādhyāya he gave the method or computing the H sines for every degree under the caption प्रतिभागज्यकाविधि

We shall now give a modern method of computing the value of rQ. From the spherical triangle RrA, where rR = 100°, $\hat{R} = i = 4\frac{1}{2}°$, we have

$$\sin 100 = \frac{\tan rA}{\tan 4\frac{1}{2}}$$

$\therefore$ log tan rA = log cos 10 + log tan $4\frac{1}{2}°$

$= 9{\cdot}9934 + 8{\cdot}8960 = 8{\cdot}8894 \quad \therefore \; rA = 4°\text{–}26'.$

From the spherical triangle rAG,

$$\cos \overline{90-\omega} = \frac{\tan rA}{\tan Gr}$$

log tan Gr = log tan rA –

log sin ω = 8·8894 – 9·6093

= 9·2801. Again from the spherical triangle rQG,

$$\cos \omega = \frac{\tan rQ}{\tan Gr}$$

$\therefore$ log tan rQ = log tan Gr + log cos ω

= 9·2801 + 9·9697 = 9·2498

$\therefore$ rQ = 10°–5′ whereas Bhāskara got 10°–22′–28″.

how Bhāskara was correct mathematically. The small difference there is, due to his taking Hsines for arcs of 15° instead of for smaller arcs.

Note. In the commentary called Sikhā of one Sri Kedāra Datta Josi (page 357) we find a mistake committed namely that he subtracted 3 R–10° the longitude of the pāta which is in Rāsis and degrees from the Sphuṭakrānti 345′–45″. What Bhāskara meant was, that since the longitude 100° exceeds 90°, the cosine will be negative which therefore entails $\triangle\beta$ to be subtracted from

Verse 7. How to know the occurrence of a pāta.

If the Sphuṭakrānti of the Moon, when it is be less than that of the Sun, then there could be no occasion for their declinations to become equal in the near future.

Comm. The situation in which the maximum declination of the Moon falls short of that of the Sun, arises when the lunar orbit comes in between the celestial Equator and the ecliptic. This situation, in its turn, depends upon the position of Rāhu. Since Rāhu's sidereal period amounts to nearly $18\frac{1}{2}$ years, the lunar orbit happens to take its position in between the celestial Equator and the Ecliptic for sufficiently a long period as shown below.

That Bhāskara could visualize this, reflects credit to his genius. This is why he formulated $\Delta\delta \pm \Delta\beta$ stressing upon the plus or minus sign.

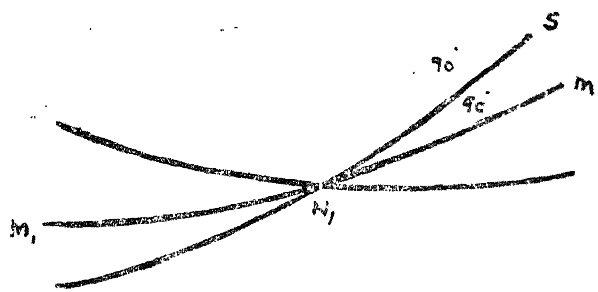


Fig. 128

Bhāskara gives an example under the above verse in the commentary to justify his finding given above. Suppose Rāhu is at the autumnal equinox and the longitudes of both the Sun and the Moon are equal to 79°. Adding the Ayanāmśas 11°, their longitudes are each 90° so that both of them are at the summer solstice ie. an Āyana Sandhi. (Of course the Moon is not exactly at the position of the Sun, but he being in his own orbit, his longitude measured along the Ecliptic is 90°). Then evidently the lunar orbit lies in between the celestial equator and the Ecliptic. Then the declination of the Moon is $24°-4\frac{1}{2}° = 19\frac{1}{2}° = 1170'$ and the declination of the Sun is $24° = 1440'$. Thus the Moon's declination when it is maximum falls short of that of the Sun. After half of the sidereal period of the Moon namely $13\frac{2}{8}$

longitudes of the Sun, Moon and Rāhu are respectively 3-2-28-12, 8-19-4-26 and 6-11-43-28 (in Rās'is, degrees etc), taking mean motions into account. When the Moon has the maximum southern declination in the position M_1 of fig. 129 his longitude will be however 8-10-9-35 and then his declination will be 1169. At that moment the declination of the Sun will be 1398. Even here the Moon's declination falls short of that of the Sun. Again after 13⅝ days, we find the declination of the Moon falling short of that of the Sun. Even after 2 months, the Moon cannot have a declination equalling that of the Sun. This phenomenon occurs when Rāhu of the lunar orbit happens to be at Libra, and the Sun is at the summer solstice approximately.

Verse 8. The definitions of Vyatipāta and Vaidhṛti.

When the Sun and the Moon are in opposite Ayanas, but in the same Goḷa, and if then their declinations be equal, then that moment is said to constitute vyatipātayoga. If on the other hand, if both the Sun and the Moon be in the same Ayana and opposite Goḷas, and if then their declinations be equal, that moment is said to constitute Vaidhṛti Yoga.

Comm. Explained before.

Verse 9 and first half of verse 10. To prognosticate the occurrence of a

When the sum of the tropical longitudes of the Sun the Moon happens to be 180° or 360°, then the Vyatipāta and the Vaidhṛti respectively will occur or will have occured. The number of minutes of arc by which the sum of the tropical longitudes falls short of or exceeds 180° or 360° as the case may be, are to be divided by the sum of the daily motions of the Sun and the Moon, which will aqproximately the number of days after which or before which the Yogas will occur or would have occured. Compute the declinations of the Sun and the Moon for that moment from the then true daily motions.

Comm. From fig. 124, it is clear that triangle rSL and $\simeq MN$ are congruent so that $rS = \simeq M$ $\therefore$ $SS_1 = MS_1$ $\therefore$ $rS + rM = \simeq M + rM = 180°$. Again triangle rSL and $rM'N'$ are congruent $\therefore$ $rS = M'r$ $\therefore$ $rS + rM' = rM' + M'r = 360°$. Thus in the former case which constitutes Vyatipāta, the sum of the tropical longitudes of the Sun and the Moon is 180°; whereas in the latter case, which constitutes the Vaidhṛtipāta, the sum of those two longitudes is equal to 360°.

Hence we are asked to note when the sum of the two longitudes is likely to amount to 180° or 360°. On a particular day suppose the sum is $180-\theta$ or $360-\theta$; so that θ is to be made up by the then velocities of the Sun and the Moon conjointly. Then $\frac{\theta}{u+v}$ where u, v are their respective velocities gives the number of days or fraction of a day, by which the sum would be 180° or 360° as the case may be. As the velocities change from moment to moment, the above $\frac{\theta}{u+v}$ is only approximate. So, compute again the respective positions and respective velocities. Suppose the sum of the longitudes is $180 \pm \theta'$ or $360 \pm \theta'$ and the velocities u' and v', Then $\frac{\theta'}{u'+v}$ gives the fraction of a day after or before the moment in question when the pātas occur Vyatipāta or Vaidhṛti.

Verse 10. To know whether the Yoga is past or future.

If the declination of the Moon situated in an odd quadrant exceeds that of the Sun or falls short of the

Sun's in an even quadraut the moment of the occurence of pāta has elapsed. Otherwise the pāta is to take place shortly after.

Comm. This is clear because in an odd quadrant, the declination of the Moon is on the increase. So if it be already greater than the Sun's, in future it will be far greater and as such cannot equal the Sun's declination. In other words the pāta had already taken place.

Similarly in an even quadrant, the declination of the Moon is on the decrease. As such if it be greater than the Sun's it will become equal in future. Thus the pāta is to take place. On the other hand if the Moon's declination is less than the Sun's, it will decrease further so that prior to the moment concerned a pāta should have occured.

Latter half of verse 11 *and verses* 12, 13, 14. To compute the time of the occurrence of pāta through a consideration of declinations.

Obtain the difference of the declinations if they be of the same direction or their sum if they be of opposite directions in the case of Vyatipāta; obtain the sum or difference of the declinations according as the declinations are of the same or different directions in the case of Vaidhṛti. Call this sum or difference 'the Ādya'. After a lapse of an arbitrary time or before the moment concerned, obtain the positions of the Sun, Moon and the Rāhu; also compute their declinations and form their difference or sum as the case may be. Call this Anya. If on both the occasions it is indicated that the pāta has elapsed or is to elapse, obtain the difference of the Ādya and Anya; otherwise their sum. Divide the arbitrary time taken, by the above sum or difference of the Ādya and Anya and multiply by the Ādya. Take the result in ghatis. Taking this as the arbitrary time, repeat the process till an invariable quantity is obtained in ghatis. This is the time by which the moment of the pāta has elapsed or after which it is going to occur.

Comm. (1) It might be asked "why bother finding the declinations at all, when it is defined that Vyatipāta happens when the sum of the longitudes of the Sun and the Moon is equal to 180° and Vaidhṛti is defined when the sum is equal to 360, since easily we could know when this happens without taking recourse to declinations?"

The problem is not that simple. The above definition in terms of the sum of the longitudes is an approximate statement, because, the original and correct definition is that their declinations must be equal in magnitude. If they be equal in magnitude but opposite in direction, they constitute Vaidhṛti Yoga. If they be both equal in magnitude and direction, the situation constitutes Vyatipātayoga. In other words, when the diurnal paths of the Sun and the Moon coincide, the moment will be Vyatipāta. If on the other hand their diurnal paths are of equal dimentions but one to the north of the celestial equator and the other to the south, then the moment is Vaidhṛti. Though the sum of the longitudes happens to be 180°, the diurnal paths may not coincide because the Moon does not actually move on the ecliptic. So the situation is more complicated.

Further the calculation of the declination of the Moon is to be done by calculating the Asphutakrānti ie. the declination of the point of the ecliptic which indicates the longitudinal position of the Moon and his celestial latitude ie. the Asphutavikṣepa from the known position of the Node, and then by rectifying the Asphutavikṣepa to obtain the Sphutavikṣepa and then adding the Asphutakrānti to the Sphutavikṣepa to obtain the Sphutakrānti.

(2) *The latter half of verse* (11) *and the first half of verse* (12).

Let us consider the case of Vyatipāta. Suppose at the moment concerned, the declinations are of opposite direction. Find the sum of their numerical magnitudes. Suppose they are of the same direction; find their difference. This sum or difference of the declinations gives the distance between the planes of the diurnal paths of the Sun and the Moon. This distance is to vanish in order that the diurnal paths may coincide. So we are asked first to find the above distance. Suppose we understand that the Vyatipātha has elapsed by noting the declinations. This may be known easily by the criteria given previously. Supposing x and y to be the declinatious of the Moon and the Sun and supposing that x is on the decrease and approaching y, then the Vyatipātha is to occur. But suppose $x < y$ and x is on the decrease, then the Vyatipāta has taken place. Thus knowing whether the Vyatipātha has elapsed or is to occur, after an arbitrary time t, compute the declinations of the Sun and Moon and form their difference or sum as the case may be which gives the distance between the diurnal paths. Let the first distance found be called Ādya and the second distance the Anya. Then the Anya will be less than the Ādya because we have taken a time towards the occurrence of Vyatipāta when the distance is to get nullified. Find the difference of the Ādya and Anya. Then by the proportion "If in time 't' taken in between the moments of the Ādya and Anya, the distance between the diurnal paths is reduced by Ādya − Anya, what time will be taken for the distance Ādya to vanish?" we have the result

$$T = \frac{\text{Ādya} \times t}{\text{Ādya} - \text{Anya}}$$ which gives approximately the time

that has to elapse for the occurrence of the pāta. This will be approximate. After a lapse of time T from the Ādya moment concerned, again compute the declinations and repeat the process. We arrive at a particular point of time, which gives the moment of occurrence of the pāta.

In the course of the commentary under these verses Bhāskara solves two problems and points out that when there is a pāta according to his exposition, the statements made by Lalla, Brahmagupta S'rīpati and Mādhava all indicate that there would not occur a pāta.

But we feel that Bhāskara read too much into those statements on account of the following. Lalla states

Brahmagupta states

मेषतुलादौ द्विभाः

चेति

and S'rīpati says

हि

In fact all these three statements mean one and the same and are intended to be approximate statements, in the first instance, having ignored the latitude of the Moon. They are just statements like that of Bhāskara himself when he says that there will be a pāta when the sum of the longitudes will be 180° or 360°. The statements purport to say that if the declination of the Moon when it is on the decrease happens to be less than that of the Sun which is on the increase, there could be no pāta. These statements so far as they go, ignoring the latitude of the Moon are perfectly in order. Bhāskara brings in a detailed analysis of two critical examples, to prove that there is a pāta, but which is negated by the rough statements of the four Ācāryas. Mādhava's statement in his Siddhānta Cūdāmaṇi is as follows which is also in similar terms as those of the other three.

रवेरोजपद्क्रान्तेः चन्द्रयुग्मपदोद्भवा
स्वल्पाचेत्तयोःक्रान्त्योः साम्यं स्यादन्यथा भवेत्

Verses 15 and 16. To obtain the duration of the pāta.

The semi-sum of the diameters of the Sun and the Moon or what is the same the sum of their angular radii being multiplied by the Spaṣṭaghatis (obtained in the estimate as per the verses 11–14) and divided by the Ādya in that context, gives the beginning and end of the pāta from the moment of the computed time. The process being repeated according to the method of successive approximations we have the correct estimate of the duration of the pāta.

Comm. This is a convention stipulated with respect to the duration of the Pātā. Strictly speaking, when the diurnal paths of the centres of the discs coincide in the case of Vyatīpāta, that will be the middle moment of the Vyatipāta. The pāta is said to last for such a time as the distance between the Centres of the discs (north-south distance) is less than $r+p$. This will be clear from a figure. The situation is akin to that of an eclipse.

The time obtained for the occurrence of the pāta under verses (11) to (14) indicates the middle of the duration of the pāta. The duration of the pāta is defined as the time that lasts as long as the declination of the highest point of one disc becomes equal to that of the lowest point of the other beginning from the moment at which the declination of the lowest point of the one becomes equal to that of the highest point of the second. In other words, just like in an eclipse, so long as the distance between the diurnal paths traced by the centres of the discs is less than the sum of their angular radii, the pāta lasts. Extending the meaning of this to Vaidhṛti also, so long as the numerical difference of the declinations of the Sun and the Moon ignoring their direction happens to fall short of the sum of the angular radii, the pāta lasts.

To obtain this duration of the pāta the argument is "If the sum or difference of the declinations according as they are of opposite or the same direction (which was taken to be Ādya under verses (11) to (14)) was reduced to zero in the time computed that time being known as Spaṣṭaghatis, what time will be taken for a difference of declinations equal to the sum of the angular radii?"

The result is $\frac{(r+p)\times T}{\bar{A}\text{dya}}$ where r and p are the angular radii of the Sun and the Moon, T the time calculated formerly known as Spaṣṭaghatis and Ādya is as defined above. The above result gives the duration of the pāta.

Note. An approximate estimate of this duration could be obtained using differentiation. We have

$$\sin\delta = \sin\lambda \sin\omega \text{ so that}$$

$$\cos\delta\, \Delta\delta = \sin\omega \cos\lambda\, \Delta\lambda$$

Let $\Delta\lambda$ be the motion in longitude of the Moon with respect to the Sun per nādi which will be on the average 12′ approximately.

$$\therefore \quad \Delta\delta = \frac{\sin\omega \cos\lambda \times 12}{\cos\delta}$$

If in one nādi, there be a variation in the declination equal to the above, what time will be taken for $16' + 15'$ the sum of the angular radii approximately? The result is

$$\frac{31 \cos\delta}{12 \sin\omega \cos\lambda}$$

When λ approaches 90°, the duration will be very much since $\cos \lambda \rightarrow 0$. This is true because the variation in the declination when $\lambda = 90°$, is very slow even for the Moon. The above formula holds good so long as λ does not approach 90°.

Verse 17. It must be deemed that the declinations will be equal so long as the difference in the declinations is less than $r + p$ numerically.

In the case of Vaidhṛti, we are asked to take the sum instead of difference and difference instead of sum in contradistinction. Why? We shall see. We know that Vaidhṛti occurs when the declinotions of the Sun and the Moon are of equal magnitude but of opposite directions subject to the condition that the longitudes are such that their sum is approximately 360°. This condition is stipulated because when the Sun is in the first quadrant and the Moon is in the 3rd quadrant and at the same time their diclinations are equal, the moment does not constitute Vaidhṛiti, because the sum of the longitudes is then less than 90° + 270° ie. less than 360°. Hence, for Vaidhṛiti one of the two celestial bodies must be in the first quadrant and the other in the fourth quadrant, so that the sum of the longitudes could equal 360° and at the same time the declinations could be equal though of opposite sign. Now compute the declinations of the Sun and the Moon at a particular moment. Suppose they are a and b and both are north. After a few ghatis or days compute again their declinations, say c and d, both again being north. Suppose then $c + d < a + b$, then Vaidhṛiti is going to occur. Construing northern declination to be positive and the southern negative, for Vaidhṛti to occur $a + b$ must be reduced to zero ie. a and b are equal and of opposite direction. Now the argument adduced is "If in time t ghatis or days $\overline{a+b}$ has become $\overline{c+d}$ (calling $a+b$ as Ādya, and $\overline{c+d}$ as Anya) there is a decrease of Ādya - Anya, time should lapse for $\overline{a+b}$ or Adya to reduce to zero?" The answer is $\frac{t\,(a+b)}{\text{Adya - Anya}}$ which gives the time after which Vaidhṛti is likely to occur.

Suppose a and b the declinations observed at a particular moment, are one north and and the other south. Compute their difference $a-b$; after a few ghatis or days again from the difference of the declinations c and d ie. $c-d$. If $c-d < a-b$ then Vaidhṛti is going to occur, otherwise it has elapsed. This is so because the difference has to vanish for Vaidhṛti to occur.

Viewing the situation algebraically, ie. construing northern declination to be positive and southern negative, we could have laid down only one criterion, instead of separating the issues and stipulating separate criteria. Proceeding on this basis, suppose the the Sun and the Moon are one in the first quadrant and the other in the second. Here both declinations are therefore positive. Compute their difference $a-b$; compute again the difference of c and d which are again northern declinations after a few ghatis or days. If $c-d < a-b$ is going to occur.

Now take a case when one of a and b is north and the other south; so also c and d. We are talking of a, c to be in the first quadrant and b, d in the fourth quadrant. Even now if $c-d < a-b$ Vyatipāta is going to occur. Thus there is no need to stipulate "Sum or difference". Difference alone counts now, because difference of one positive and the other negative quantities means sum only. Thus in the case of Vyatipāta difference should tend to zero, whether both the declinations are north, or one north and the other south.

Similarly in the case of Vaidhṛti the sum should tend to zero because the algebraic sum of one positive northern declination and the other negative southern declination shall be zero, if the declinations were to be equal and of opposite directions.

Here ends the Pātādhyāya.

Note Bhāskara gives two examples in the course of the commentary, one, to show the fallacy in asserting that there would be a pāta when the Sun is in an even quadrant, the Moon in an odd quadrant and the Moon's declination is less than that of the Sun; and the other to show the fallacy in asserting that the pāta has elapsed when it is still to take place, and that it is still to occur when it has already elapsed. He quotes the example given in his Golādhyāya wherein the longitudes of the Sun, Moon and Rāhu are 120°, 60° and 180° respectively, there being no Ayamas'as. (Since at the time of Lalla, there were no Ayanams'as, Bhāskara gives such an example), Since the Sun is in an even quadrant and the Moon in an odd quadrant, Bhāskara says, that as per the verse of Lalla 'सूर्यापमात् ओऽपक्रोद्भवात् etc.' there should be a pāta. But since the longitude of Rāhu is 180°, Ketu is at the equinoctial point so that the lunar orbit lies between the ecliptic and the equator. Hence the declination of the Moon remains smaller than that of the Sun for a good length of time never equalling the declination of the Sun. This, Bhāskara says, is a fallacy regarding 'भावाभावत्व' ie. 'Is there a pāta or not' because when there is actually no pāta, Lalla's criterion leads to assert that there is one. It may be reiterated here, that Lalla gave a general criterion ignoring the latitude of the Moon ie. supposing the Moon to be moving on the ecliptic. Bhāskara takes the latitude also into consideration and proves that there is no pāta.

In the second example, which Bhāskara gives, he tries to show the fallacy with respect to "गतैष्यत्व" ie. asserting that a pāta has elapsed when it is still to occur, and that it is to occur when it has already elapsed.

In this example the declinations of the Sun and Moon are respectively 1416′, 1324′, when their tropical longitudes are 80° and 100°. The Moon is in an even quadrant, and his declination is less than that of the Sun. As per the criterion of Lalla given in the verse "सूर्यपमात् etc." the pāta is not there at all. But Bhāskara computes and shows that the pāta has occured 70 nādis before (Rāhu having a reverse tropical longitude of 100°). We shall shew here, how using differential calculus we could cut short the process. We have

$$\sin \delta = \sin \lambda \sin \omega \text{; differentiating}$$

$$\cos \delta \, \triangle\delta = \cos \lambda \, \delta\lambda \sin \omega \text{ or}$$

$$\triangle\delta = \frac{\sin \omega \cos \lambda}{\cos \delta} \triangle\lambda$$

$$= \text{Āyanavalana} \times \text{Variation in Bhuja.}$$

This could be seen graphically also from fig. 129. Let P and Q be two contiguous positions of the Sun (say) on the ecliptic when arc $pQ = \triangle\lambda$. Let the increment in his

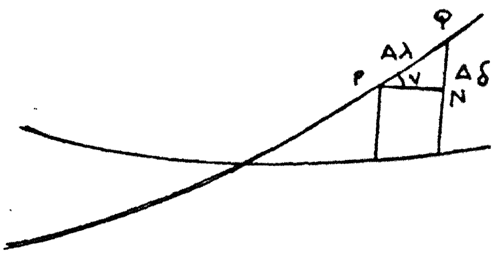


Fig. 129

declination in going from P to Q be NQ equal to Taking PNQ to be a plane $\triangle$ $\triangle\delta = \triangle\lambda \sin v$ where

$v = QPN$. We have named this angle to be v, because it is no other than the Āyanavalana which was formulated to be equal to $\frac{\sin \omega \cos \lambda}{\cos}$. Thus $\triangle\delta = \triangle\lambda \times$ valanajyā.

APPENDIX

LIST OF TECHNICAL TERMS

1. *Adhikamasa* : A month gained by the lunar reckoning over the solar. This is located in that lunar month which does not contain a

2. *Agrajya* : The Hindu sine of the arc of the horizon in between the rising point of the Sun and the east point.

3. *Akṣa (or) Pala* : Latitude (Terrestrial)

4. *Akṣa Dṛkkarma* : The arc of the ecliptic between the point of intersection of the ecliptic with a secondary through the star to the prime vertical and the point of intersection of the ecliptic with the star's declination circle.

5. *Akṣakarṇa* : The hypotenuse of the gnomonic triangle when its shadow is equal to what is called

6. *Akṣa-Kṣetra* : A right-angled spherical triangle one of whose sides may be the arc of a small circle namely the diurnal circle but one of whose angles is a right angle and another equal to the terrestrial latitude.

7. *Akṣavalanam* : The angle at the point of the star in between the declination circle of the star and a secondary to the prime vertical through the star.

8. *Antyā* : The Hindu sine of an arc of the celestial equator corresponding to *Hṛti*.

9 *Asta* : Setting or heliacal setting.

10. *Ayanabindu* : Solstice.

11. *Āyana-Dṛkkarma*: The arc of the ecliptic intercepted between its point of intersection with the star's declination circle and the secondary to the ecliptic through the star.

12. *Ayanaṁśam* The arc of the ecliptic in between the vernal equinoctial point and the Hindu zero of the ecliptic ie the first point of the Zodiacal sign called *Aśvinī*.

13. *Āyanavalanam* The angle at the point of a star, between its declination circle and the secondary to the ecliptic through the star.

14. *Bārhaspatyamāna*: The time taken by Jupiter to reside in a *Rāśi*, on the average, is called a jovian year. This falls short of a solar year.

15. *Bhāga* : A degree

16. *Cāpa or Kārmuka* : Arc.

17. *Carajyā* : The Hindu sine of the arc intercepted between the east point and the declination circle of a rising star or planet or the Sun.

18. *Cāndra-māsa* : The time between two consecutive full-moons or New moons.

19. *Chāyā or Bhā* : Shadow cast by the gnomon.

20. *Chāyābhuja* : The projection of the shadow on the east-west line.

21. *Chāyākarṇa or Bhākarṇa* : The hypotenuse of the gnomonic triangle whose two sides are the gnomon and its shadow.

22.	*Chāyākoṭi*	: The perpendicular from the extremity of a shadow on the east–west line.
23.	*Dhruva*	: The star near the celestial pole or the celestial pole itself.
24.	*Dhruvaka*	: The celestial longitude.
25.	*Dhruva–protavṛttam*	: The declination circle.
26.		: The Hindu sine of the azimuth measured by the angle between the prime vertical and the vertical of a star or a planet.
27.	*Dorjyā or Bhujajyā*	: Hindu sine of celestial longitude.
28.	*Dṛgjyā*	: The Hindu sine of the Zenith distance.
29.	*Dṛg-lambana*	: Total parallax.
.0.	*Dvāparayuga*	: Twice the period of a kaliyuga.
31.	*Dyujyā*	: The Hindu cosine of declination or the radius of the diurnal circle taking the radius of the celestial equator to be R equal to 3438 units.
32.	*Dyujyā-vṛtta or Ahorātra-vṛtta*	: The diurnal circle of a star or a planet.
33.	*Ghaṭi or Nāḍi*	: An interval of time equal to 24.' (minutes)
34	*Grahaṇa*	: Eclipse.
35.	*Iṛti or Iṣṭahṛti*	: The Hindu sine of the arc of the diurnal circle from a point of the same upto the plane of the horizon.
36.	*Kadamba*	: Pole of the ecliptic.
37.	*Kadamba-protavṛtta*	: A secondary to the ecliptic through a star or planet.

38. *Kakṣāmaṇḍala* : The deferent of a planet or the circle with the earth as centre and radius equal to 3438 units.

39. *Kaḷā* : The Hindu sine in the diurnal circle corresponding to the *Sūtra* (given below).

40 *Kalā or Liptā* : A minute of angle.

41. *Kaliyuga* : The period consisting of 4,32,000 mean solar years.

42. *Kalpa* : *Dvāparayuga* is twice *Kaliyuga*; *Tretāyuga* thrice and *Kṛta* four times. All these put together constitute a *Mahāyuga.* 71 *Mahāyugas* make one *Manvantara.* Fourteen Manvantaras with what are called Sandhi periods on either side equal to a *Kṛtayuga* or thousand *Mahāyugas* make a *Kalpa.* The creation is supposed to last one *Kalpa*, which is said to constitute the day time of *Brahma*, the Creator. His night also is of the same duration when there is no creation. It is held that we are now in what is called *Śveta-Varāha Kalpa,* wherein the seventh Manvantara called *Vaivasvata manvantara* is current. In this *Manvantara*, twenty seven *Mahāyugas* are supposed to have elapsed and in the twenty-eighth *Mahāyuga, kṛta Treta* and *Dvāparayugas* have elapsed and that we are now in the *Kaliyuga.* In this *Kaliyuga* which began on the day when Lord *Kṛṣṇa* gave up his mortal coil, 5080 mean solar years have elapsed by about 21 March, 1979.

43. *Kramajyā or simply jyā or Jīvā or guṇa* : Hindu sine of an angle.

44. *Karaṇa* : Half of the duration of a *tithi.*

45.	*Karṇāgrajyā or simply Karṇāgrā*	: The Hindu sine *Agrajyā* multiplied by K and divided by R where K is the hypotenuse of the gnomonic triangle whose two sides are the gnomon and its shadow and R the radius of the celestial sphere taken to be 3438 units.
46.	*Kendra*	: The centre of a circle.
47.	*Ketu*	: The diametrically opposite point of *Rāhu*. *Rāhu* also means the circular section of the earth's shadow at the Moon.
48.	*Kha-Svastika*	: The Zenith at a place.
49.	*Koṭijyā*	: Hindu Cosine of an angle.
50.	*Krānti or Apama*	: The declination of a point on the ecliptic.
51.	*Krānti-Vṛttam*	: Ecliptic.
52.	*Kṛtayuga*	: Four times the period of a *Kaliyuga*.
53.	*Kṣayamāsa*	: That lunar month in which there are two *Saṁkramaṇas*.
54.	*Kṣitija*	: The Horizon at a place.
55.	*Kujyā*	: The Hindu sine measured in the diurnal circle corresponding to the *Carajyā* defined above.
56.	*Lagna*	: The *Rāśi* which rises at any moment or the rising point of the ecliptic.
57.	*Laṁbajyā*	: The Hindu Cosine of the latitude.
58.	*Laṁbana*	: Parallax in longitude.
59.	*Mahāyuga*	: The Sum of the four *yugas* mentioned above.
60.	*Manvantara*	: A period equal to 71 *Mahāyugas*.

1'. *Nakṣatra* : A star. Also the time which elapses as the longitude of the Moon increases by $13\frac{1}{3}$ degrees starting from the zero point of *Aśvinī*.

62. *Nākṣatra-Dina* : The time that elapses between two consecutive risings of a star.

63. *Nākṣatra-māsa* : The time taken by the Moon to go from *Aśvinī* again to *Aśvinī*.

64. *Natakāla* : Hour angle measured in *ghaṭis*.

65. *Natāṁśajyā* : Hindu sine of the Zenith distance.

66. *Nati* : Parallax in latitude.

67. *Nīcoccavṛtta* : Epicycle.

68. *Parama-Antyā* : The *Antyā* when the celestial body is at the point of intersection of the celestial equator and the meridian or what is the same at the point of culmination.

69. *Paramakrānti* : Obliquity of the ecliptic taken by the Hindu astronomers to be 24°.

70. : The point of time when the declinations of the Sun and the Moon are equal and of the same sign or opposite sign. Also it means the point of intersection of two great circles.

71. *Prācī* : East point.

72. *Prācyaparā* : East-west line.

73. *Prativṛtta* : The circle in which the planet moves and whose centre is at a distance from the centre of the Kakṣāmandala or the orbit of the mean planet. This is the same as the orbit of the the true planet supposed to be a circle.

Rāhu : The point of intersection of the Moon's path with the ecliptic (ascending point of the Moon's path). Also it means the circular section of the earth's shadow at the Moon.

75. *Rāśi* : An arc equal to 30° (on the ecliptic).

76. *Samabindu or Udagbindu* : North point.

77. *Samamaṇḍala* : Prime vertical.

78. -*Śaṅku* : The Hindu cosine of the altitude when the Sun is on the Prime Vertical.

79. : The point of time when the Sun enters from one *Rāśi* to another of the twelve *Raśis, Meṣa* etc. measured from the zero-point of the Hindu Zodiac.

80. *Śaṅku* : Gnomon.

81. *Śaṅku (Also means)* : The Hindu sine of the Altitude.

82. *Śaṅku-cchāyā* : The shadow cast by the gnomon.

3. *Saura-māsa* : The time when the Sun occupies one *Rāśi*.

84. : The Hindu versed-sine of the hour-angle. Also it means celestial latitude.

85. *Sāvanāha* : The time between two consecutive Sun-rises.

86. *Sūtra* : The Hindu sine of the complement of the hour-angle.

87. *Taddhṛti* : The *Hṛti* of a celestial body when it is on the prime vertical.

88. *Tithi* : The time taken by the elongation of the moon to increase by 12° starting from zero.

89. *Tithi-kṣaya* : Since a *Tithi* falls short of a Sāvanāha or civil day, in course of time, it so happens, that a tithi begins after sun-rise and does not last upto the next suu-rise. Such a tithi is said to be lost and goes by the name *Tithi kṣaya.* One such tithi kṣaya occurs out of 64 tithis approximately or eleven out of 703 more approximately.

90. *Tretāyuga* : Thrice the period of a *Kaliyuga.*

91. *Trijyā* : The Hindu sine of three Rasis or 90° equal to R or 3438 units.

92. *Udaya* : Rising or heliacal rising.

93. *Unmaṇḍala or Udvṛtta* : The great circle through the celestial pole and the east point.

94. *Unmaṇḍala Śaṅku* : The Hindu cosine of the altitude of a celestial body situated on the *unmaṇḍala.*

95. *Upavṛtta* : A small circle parallel to the prime vertical through a star or a planet.

96. *Vighaṭī or Vināḍī*: One sixtieth of a *ghaṭi.*

97. *Vikṣepa or śara or Viśikha* : Celestial latitude.

98. *Viṣuvatbindu* : Equinoctial point.

99. *Viṣuvat-chāyā* : The shadow cast by the gnomon when the Sun is at the point of intersection of the celestial equator and the meridian on an equinoctrial day.

100. *Viṣuvat-Vṛtta* : Celestial equator.

101. *Vṛtta or Maṇḍala*: A circle.

102. *Yāmyottara-saṁku or* — The Hindu sine of the Altitude of a Celestial body situated on the meridian.

103. *Yāmyottaravṛtta* : The meridian at a place.

104. *Yaṣṭi* — $R^2 - Ayanavalanajā^2$

Yaṣṭi has also another meaning namely the length of the perpendicular from a point on the diurnal circle on the plane parallel to the plane of the horizon through the point of intersection of the diurnal circle with the *unmaṇḍala.*

105. *Yoga* — The time which elapses when the sum of the longitudes of the Sun and the Moon to increase by $13\frac{1}{3}^{\circ}$ starting from zero.

106. *Yuti* : Conjunction.